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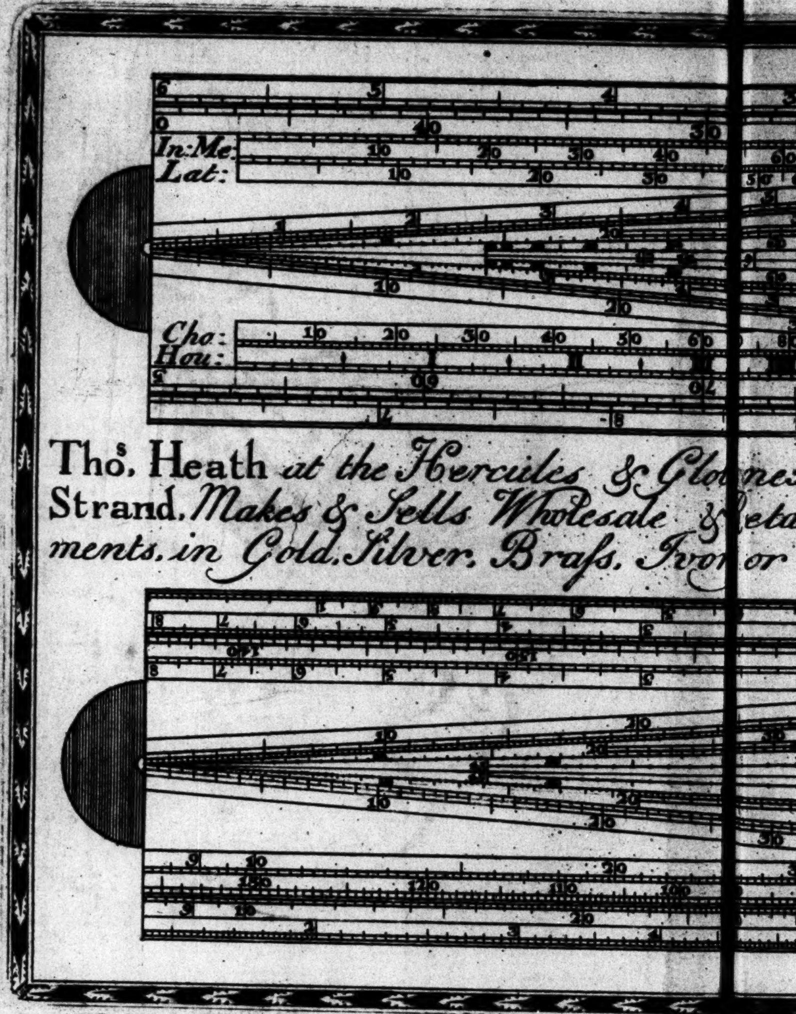
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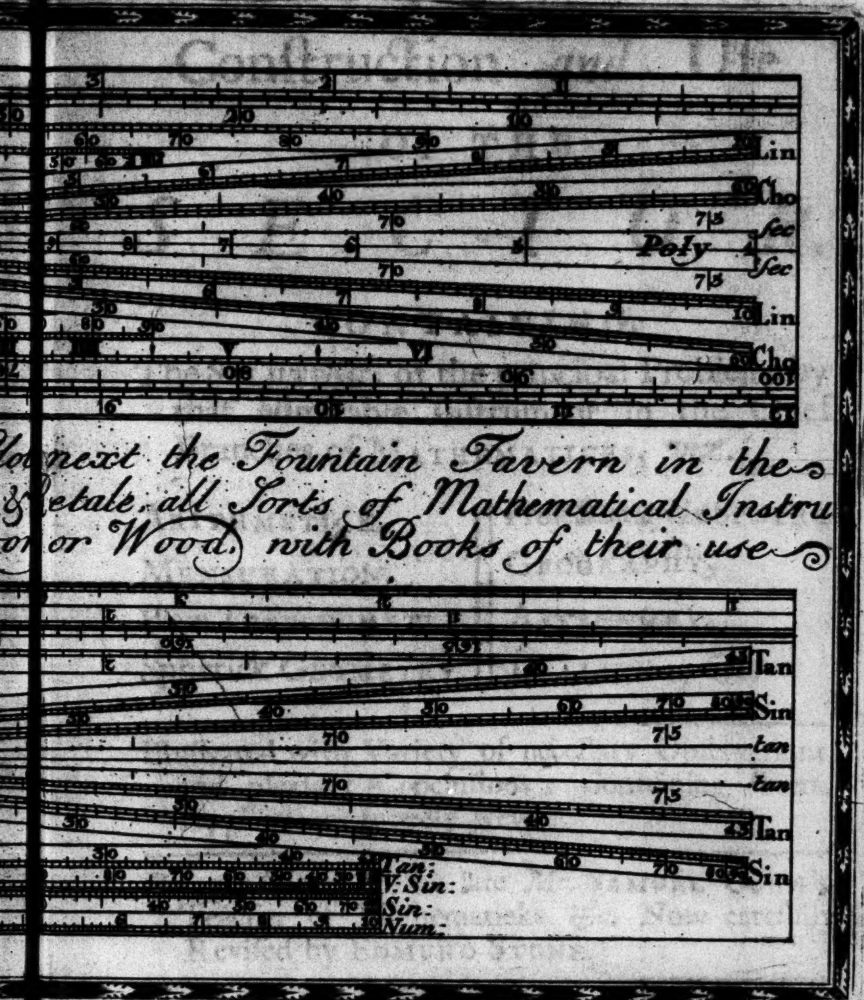
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A NEW

TREATISE

OF THE

Conſtruction *and* Uſe

OF THE

SECTOR.

CONTAINING,

The SOLUTIONS of the principal Problems by that admirable Inſtrument in the Chief Branches of MATHEMATICKS, *viz.*

ARITHMETICK,	Projection *of the* SPHERE
MENSURATION,	GEOGRAPHY,
Plain TRIGONOMETRY,	ASTRONOMY,
Spherick GEOMETRY,	DIALLING, &c.

Illuſtrated with Variety of neceſſary Obſervations, and pleaſant Concluſions: Containing ſeveral Applications intirely New.

Being a Work of the late *Mr.* SAMUEL CUNN's, Teacher of Mathematicks, &c. Now carefully Reviſed by EDMUND STONE.

LONDON:

Printed for JOHN WILCOX, at the *Green Dragon*, in *Little Britain*, and THOMAS HEATH, Mathematical Inſtrument Maker, at the *Hercules* and *Globe*, next the *Fountain Tavern* in the *Strand.* M.DCC.XXIX.

THE REVISOR TO THE READER.

AMONG the Multitude of Mathematical Instruments that have been invented, the Sector *(as we call it) or,* Compaſs of Proportion *(as the* Foreigners *) claims a principal Place, and, ever ſince the Invention thereof, has been had always in the greateſt Eſteem by the Ingenious of* the Mathematical Kind, *but more eſpecially with thoſe who buſy themſelves in the practical Parts of that* Learning.

The Value of this Inſtrument *may be eaſily gathered from hence; That ſeveral eminent* Mathematicians, *as well* Engliſh *as* Foreigners, *have thought it worth their Pains to write Treatiſes concerning it. Such as* Juſtus Burgius, *and* Goldman, *in*

Latin; Galileo, *in* Italian; Dechales, *in* Latin; Ozanam, *in* French; Gunter, Foster, *&c. in* English; *And now, at length, the late Mr.* Samuel Cunn, *a Person whom all Men, capable of judging, must allow, in justice, to have been a* Mathematician *of the first Rank, and consequently would never have written now on a Subject that was trifling, or well handled by Others before.*

The Sectors *made formerly (as well as the Foreign ones now) differ very much from those made by our Instrument Makers at this Time. I mean as to the Scales of Lines drawn upon the Face of them, and their Positions; These latter having several Lines omitted, viz.* The Lines of Quadrature, the Lines of Segments and Inscribed Bodies, the Lines of Planes, the Lines of Solids, the Lines of Metals, *&c. to make Room for others of far more extensive Use; and the Position of those (that issue from the Center) are so alter'd, that the* Sector, *as now made, is not only a Scale of* Chords, Sines, Tangents, *&c. to innumerable Radius's, but likewise by it you may readily work Proportions in* Lines, Sines, Tangents, *&c. separately, or with each other, and that to an Exactness sufficient enough for ordinary Practice, if the Instrnment be tolerably large.*

From

From whence it manifestly appears, that this Instrument is vastly useful in Trigonometry, Spherical Geometry, Projection of the Sphere, Practical Astronomy, Dialling, *and, indeed, all* Practical Parts *of* Mathematicks *where Scales of equal Parts,* Chords, Sines, Tangents, *&c. are concerned.*

Now Mr. CUNN *having examin'd the Divisions on several* Sectors, *even Brass ones of* 12 *Inches, made by notable Workmen, found egregious Faults in all of them; upon which he* himself, *with Mr.* Heath, *Mathematical Instrument Maker, in the* Strand, *took the Pains of dividing anew from* Sherwin's *Tables, a Brass Pattern of a* Foot Sector.

Hence, if we consider Mr. CUNN's *Skill and Accuracy in Things of this Nature, it will evidently follow, that* Sectors *carefully made by the said Pattern, must equal, or even exceed, any others whatsoever, in answering the End designed.*

This Faultiness of Sectors *was the principal Cause of Mr.* CUNN's *writing the following little Tract of the* Foot Sector, *which will serve for any sized ones, wherein he has profoundly handled the Matter, and not so much as omitted one single Circumstance of Use, in giving the Reader a thorow*

Know-

Knowledge of this admirable Instrument, both in its Constraction and Use.

And, to compleat the Whole, tho' to me there seems no absolute Necessity, he has given the Construction and Uses of the Artificial Numbers, Sines, *and* Tangents, *which are Lines properly belonging to* Gunter's Scale, *but now put upon* Sectors, *as well to fill up vacant Spaces, as for their good Uses. He gives you likewise the Construction and Uses of the Dialing* Lines of Latitudes *and* Hours.

But, that he might not be altogether silent with respect to the Lines formerly put upon Sectors, *he at length finishes the Piece with a brief and distinct Account of their Nature and Use.*

Thus much in general; Not in the least doubting but the following Sheets will be read with abundance of Pleasure by every Practical Mathematician.

Edmund Stone.

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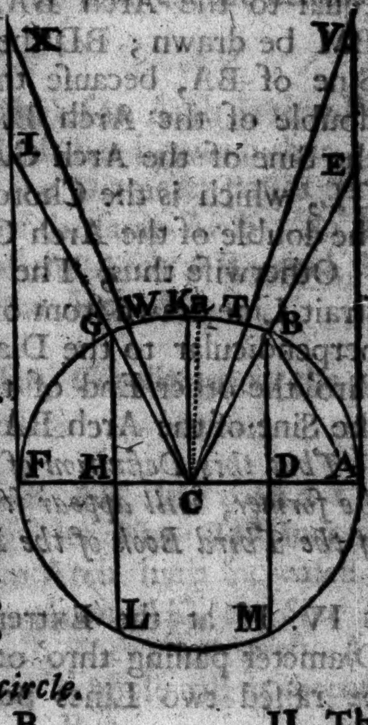
OF THE

LINES *upon the* SECTOR.

CHAP. I.

Chords, Sines, Tangents, *&c.* *defined.*

I. ANY Part of the Circumference of a Circle is called an *Arch*; so either AB or AG (*Fig.* 1.) AGL or AGLM is an Arch: But the Arch AK, because the fourth Part of the Circumference or Periphery, is called a *Quadrant*; and the Arch AKF is called a *Semi-circumference* and sometimes, tho' improperly, a *Semicircle*.



II. The ſtrait Line which is drawn from one End of an Arch to the other End of the ſame, is call'd the *Chord* of that Arch; ſo BM is the Chord of the Arch BAM, and alſo the Chord of the Arch BFM, ſince both theſe Arches have the ſame Ends, and the ſtrait Line BM is drawn thro' thoſe Ends, and terminated by them. In like manner the ſtrait Line BA is the Chord of the Arch BA, and alſo of the Arch BFLA, and GL the Chord of GAL and of GFL.

III. The Right Line, or (for its great Uſes) the *Sine* of an Arch, is half the Chord of double that Arch; ſo if AM be taken equal to the Arch BA, and the ſtrait Line BM be drawn; BD the half of BM is the Sine of BA, becauſe the Arch BAM is the double of the Arch BA. In like manner, the Sine of the Arch GA is GH, the half of GL, which is the Chord of the Arch GAL, the double of the Arch GA.

Otherwiſe thus, The *Sine* of an Arch is a ſtrait Line drawn from one End of that Arch, perpendicular to the Diameter which paſſes thro' the other End of that Arch; ſo BD is the Sine of the Arch BA.

That this Definition of a Sine co-incides with the former, will appear from the 3d Propoſition of the Third Book of the Elements of Euclid.

IV. If at the Extremities of a Circle's Diameter paſſing thro' one End of an Arch, be raiſed two Lines perpendicular to that Diameter;

Diameter; those two Lines will touch the Circle in those Extremities, by the 16th of *Euclid's* 3d Book; and if from the Center of the Circle be drawn a strait Line through the other End of the forementioned Arch, and this strait Line be produc'd till it meets one of the foresaid Perpendiculars; then that Part of the Perpendicular which is between the Point of meeting and the Diameter, is called the *Tangent* of that Arch. And that Part of that Line which meets the Tangent, and limits it; I say, that Part, which is between the Center and the Point where it meets the Tangent, is called the *Secant* of that Arch; so AE is the Tangent, and CE the Secant of the Arch BA, and FI is the Tangent, and CI the Secant of the Arch FG.

V. The *versed Sine* of an Arch is that Part of the Diameter which lies between that End of the Arch which the Diameter passes thro', and the right Sine of the same Arch. So AD is the versed Sine of the Arch AB; and AH the versed Sine of the Arch AG.

VI. Some necessary Corollaries to the preceding Definitions.

1. While an Arch AB, less than a Quadrant is increasing, its Sine BD increases also: But when the Arch hath increased till it is become the Quadrant AK, the Sine KC is the greatest of all Sines; for if the Arch should still increase, suppose till it becomes

AG, its Sine GH is less than KC. (These Things are evident from the 3d and 15th Props. of *Eucl. lib.* 3.) that is, after the Arch is become a Quadrant, if it continues to increase, the Sines will continually decrease, even till the Arch becomes AKF.

2. Since KC the Quadrant's Sine falls on the Center of the Circle; KC is a Radius of that Circle, and is equal to CA or CF, and is sometimes called the *whole Sine*.

3. If the Arch FG be equal to the Arch AB, the Sine of the former GH, is equal to BD, the Sine of the latter. But when the Arch FB is equal to the Arch AG, the Sine of the former BD, is equal to GH the Sine of the latter: (These Things are evident from the forementioned 3d and 15th Props. of *Eucl. lib.* 3.) And hence it appears, that the 4 Arches AG, FG, AB, FB, have equal Sines GH, BD; and of these the two former have one and the same Sine GH; and the two latter one and the same Sine BD. Hence it appears, that any Arch AG, and the Arch FG, which is its Supplement to a Semicircle, have the same Sine.

4. While an Arch AB increases to AT, the Tangent AE will be increased to AV, and the Secant CE to CV. And if an Arch less than a Quadrant continues to increase till it becomes the Quadrant AK, the Secant Line, or rather that which should be the Secant Line, falls on KC, which is perpendicular to AC, as well as AE; and so KC (28 p. of *Euc. lib.* 1.) is parallel to AE; consequently that Line which should be the

Secant to determine the Length of the Tangent and itself, doth not meet it. Therefore in this Case, that is, when the Arch is a Quadrant, the Tangent and Secant are both infinite. But if the Arch AB had so increased as to want of being equal to AK, a very small Quantity *a* K; the Secant Line C *a* produc'd would meet the Tangent, and determine that and itself: but at a Distance very great. And so AB continually increasing till it becomes *a* A; the Tangent and Secant continually increase likewise.

5. The Arch being greater than AK, and continually increasing, the Tangent and Secant are continually decreasing, even till the Arch becomes AKF; for the Tangent of the Arch AW is FX, and its Secant is CX; but the Tangent of the Arch AG is FI, and its Secant is CI. But FI is evidently less than FX and CI (24 Prop. of *Euc. lib.* 1.) is less than CX.

6. In the Tangents and Secants, as in the Sines, when two Arches together are just a Semi-circumference, they have the same Tangents, and the same Secants.

7. If the Arch AB increases, the versed Sine AD increases; if the Arch AB increases to the Quadrant AK, the versed Sine becomes AC the Radius; if it still increases, till it becomes AG, the versed Sine becomes AH: So that the versed Sine always increases with the Arch, even when the Arch is greater than a Quadrant, until the Arch become equal to AKF. Therefore the same versed Sine doth not

anſwer to an Arch, and its Supplement to a Semi-circumference.

CHAP. II.

The Lines of Chords, Sines, Tangents, *&c. defined.*

1. IN order to theſe Definitions, we are to obſerve, that the Circumference of a Circle is commonly divided into 360 equal Parts, called *Degrees*; and theſe again divided, or ſuppoſed to be divided, into 60 equal Parts, called *Minutes*; and theſe again are ſubdivided continually by 60, if the Circumference be large enough.

2. And when the Length of an Arch contains any Number of theſe Degrees, it is called an Arch of thoſe Degrees; ſo if an Arch contains 57 of ſo many Degrees, it is call'd an Arch of 57 Degrees.

3. And the Chord, Sine, Tangent, or Secant of that Arch, is called the Chord, Sine, Tangent, or Secant of (57) the Degrees of that Arch.

4. Now a *Line of Sines* is a Line to which the Sines of every Degree of the Quadrant are transferred and number'd, as their correſponding Arches of the Circle, and all of them

them beginning from one Point. Such a Line is CG in Fig. 2. In that Line C 20, is the Sine of 20; C 50 the Sine of 50; and CG the Sine of 90, to the Radius CF or CA.

5. What hath been said of a Line of Sines may be applied to a Line of *Chords*; *viz.* It is a Line to which the Chords of every Degree, as far as 60, 90, 180, your designed Length, are transferred. All of them begin from the same End of that Line; and the other Ends of these Chords so transferr'd are number'd according to their Correspondent Arches. Such Lines are AE, AD, Fig. 2.

6. Lines of *Secants* and *Tangents* are Lines to which as many of the Secants and Tangents are transferred, as the Length of the Instrument will receive, and number'd as noted above. So FH, Fig. 2. is a Line of Tangents to 70; and if the Divisions on CG were struck out, CK would be a Line of Secants to 70 Degrees.

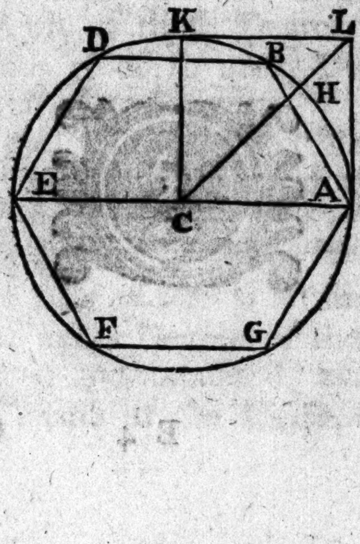
CHAP. III.

Of the Radius *on the* Chords, Sines, Tangents, *and* Secants.

1. IT hath been shewn in the first Chapter (Coroll. 2.) that the Sine of 90 is equal to the Radius of the Circle, and therefore may at all Times be taken for it, and called by the same Name.

2. A Side of an Equilateral 6 sided Figure, inscribed in a Circle (that is a Chord of the 6th Part of the Circumference, or of 60 Degrees) is equal to the Radius of that Circle (15 Prop. of Book IV. of *Euclid*).

Of the equal Sides of the *Hexagon* (Fig. 3.) any one of them AB, or BD, or DE, *&c.* is



the Chord of 60 Degrees, and equal to the Radius of the Circle CA.

3. The Tangent of 45 Degrees is alſo equal to the Radius of the Circle, and may therefore be taken for it.

For in the ſame Figure, raiſe CK perpendicular to CA, and KL perpendicular to CK, and produce it till it meets AL. Then (28 Prop. of *Eucl. lib.* 1.) is AL parallel to CK, and KL to CA; and ſo ACKL is a Parallelogram: It is alſo a Square, ſince KC is equal to CA. Whence (34 Prop. *Euclid, lib.* 1.) KL is equal to CA, and LA to KC, draw CL; then by (8 Prop. *Euclid, lib.* 1.) the Angle KCL is equal to the Angle ACL; and ſo (26 Prop. *Euclid, lib.* 3.) the Arch KH is equal to the Arch HA; that is, HA is an Arch of 45 Degrees, ſince KA is one of 90 Degrees. LA, which hath been ſhewn to be equal to CA, is the Tangent of the Arch HA; therefore LA, the Tangent of 45 Degrees, is equal to CA the Radius, and therefore may be taken for it.

4. The Secant of 0 Degrees is equal to the Radius, and therefore may be taken for it.

For (in Fig. 1.) if the Arch BA decreaſes, the Secant CE will decreaſe alſo. Let the Arch decreaſe to nothing, that is, let B come to A; then will E come to A, and CE coincide with CA. Therefore the Secant of 0 Degrees may be taken for the Radius.

CHAP.

CHAP. IV.

The Geometrical *Construction of the Lines of* Chords, Sines, Tangents, *&c.*

FROM *the Second Chapter, it appears, That the first Thing necessary to perform the Work proposed, is a Circle divided into 360 equal Parts exactly. But this Task is so hard, that common Geometry pronounces it impossible. It is true, indeed,* Euclid *in his 4th Book hath shewn us how to divide the Circumference of a Circle into* 3, 4, 6, 5, 10, 15, &c. *equal Parts; and how from them to gain other equal Parts: And yet all these, and all those that may be deduced from them, are insufficient for our Purpose.*

Dr. Halley *(I suppose) seeing the Impossibility of performing this Division of the whole Circumference into 360 equal Parts Geometrically,* did, *indeed,* divide *his large Mural Quadrant of* 8 *Feet Radius into* 90 *Parts; but as a Check to these* 90 *Divisions, adds another Number of Divisions;* viz. 96. *which might be made equal Parts by common Geometry. And then by a Table of Reduction compares the two Orders of Divisions.*

The Design in this Chapter being to illustrate fully and Geometrically what these Lines are; I shall

I shall here take it for granted that a Circle is so divided, and leave the Manner of attaining it by Approximation *for a while.*

A Circle then, being quartered, divided, and number'd (as in Fig. 2.) I proceed, and first for the Chords.

1. Draw AD. Then setting one Foot of the Compasses in A, extend the other to 10 on the Arch; and with that Distance, the Compasses still resting in A, describe the Arch 10, 10, cutting the Line AD in 10: And in like manner, one Foot still resting in A, extend the other on the Arch successively to the Points;

$$\left\{\begin{matrix}20\\30\\40\\50\\60\\\&c.\end{matrix}\right\}\text{ and describe the Arches }\left\{\begin{matrix}20,\ 20\\30,\ 30\\40,\ 40\\50,\ 50\\60,\ 60\\\&c.\ \&c.\end{matrix}\right\}\text{ cutting the}$$

$$\text{Line AD in the Points}\left\{\begin{matrix}20\\30\\40\\50\\60\\\&c.\end{matrix}\right\}.\quad\text{Do the like}$$

with the intermediate Divisions of the Arch, and number the Divisions on the right Line AD, so that they may correspond with the Numbers on the Arch, and you will have a Line of Chords to 90 Degrees. If you want it but to 60, you need not lay any Divisions

on

on the Line beyond 60, or instead of AD you might have used AE.

2. Now for the Tangents draw from the Center C, strait Lines to the Divisions in the Quadrant GF, and produce them till they meet the Perpendicular FH; so shall FH be a Line of Tangents divided, and you must continue it to as many Degrees as will come on the assigned Length. These must be number'd correspondent to the Arches.

3. For the Secants: Produce CG towards K; then setting one Foot of the Compasses in C, extend the other to the several Divisions in the Tangent Line, and describe Arches meeting GK; and the Line CK will be a Line of Secants divided between G and K, which must be number'd as the Line of Tangents is, from whence the Arches forming the Divisions were described. This Line begins at C, and must be continued to as many Degrees as the Length assign'd will permit.

Here observe, that the smallest Secant is greater than the Radius; and that therefore there are no Divisions between C and G.

4. For the Sines. From the several Divisions in the Quadrant FG, let fall Perpendiculars to CF; and those Perpendiculars may be transferred to one Line, and numbered to answer the respective Divisions on the Arch, and it will be a Line of Sines.

Because the greatest Sine is CG; and because there are no Divisions on the Secant Line CK between C and G: if those Sines are transferr'd to CG; you have the Line of

Sines

Sines as you there see. And this is the way they are usually laid down on *Gunter's* Scale.

5. By the preceding Work you have divided the Line FC; which, if number'd as you there see, is a Line of versed Sines, as far as a Quadrant or 90 Degrees. And if the like be perform'd in the other Quadrant CAG; or if C 100 be made equal to 80, C 110 equal to C 70, &c. and so on to 180; AF will be a whole Line of versed Sines.

6. But the Line of Sines may be otherwise constructed thus. The two Quadrants AG, FG, being divided and number'd from A and F towards G; draw Lines from 10 to 10, from 20 to 20, from 30 to 30, &c. and also from the intermediate Divisions; and the Line CG will be a Line of Sines divided, which must be number'd according to the Numbers on the correspondent Arches.

In the next Place, to lay down these Lines on the Sector, by the help of Mathematical Tables, carefully calculated or corrected; the Geometrical way by the oblique Intersections of Lines being not sufficiently exact;

I shall take for an Example, a Sector whose Legs were each one Foot. I mention the Length on account of the Number of the smaller Divisions between the principal ones. I chuse this Sector for an Example, because I have tried occasionally Brass ones of the same Size, the Works of several notable Workmen, but could find none without egregious Faults. As to the Accuracy of this, I shall refer to the Gentlemen who will be pleased to consider the

Methods

Methods used in dividing it, to try it and compare it with others.

CHAP. V.

To lay down the Sines, Tangents, *&c. by the help of Tables.*

THE first Thing absolutely necessary to this Work, is a Line of equal Parts; or at least of Parts so nearly equal, that the Differences from Equality may be insensible.

Mr. Edm. Gunter, *formerly Professor of* Astronomy, *in* Gresham College, London, *lays so great a Stress on this Line, that he says it is the Ground of all the rest. In his Treatise of the Making of the Sector, he gives calculated Tables for the laying down every Division on his Sector, by a Line of equal Parts.*

Mr. Samuel Foster, *his Successor in the Professorship, treating of the Construction of the Lines both Natural and Artificial, shews how to perform the Work by a Line divided into equal Parts, and calculated Tables; he never so much as once mention'd the Performance of any one by the Intersections of Lines. From whence, by their Judgments it appears, that this is the only Method to be depended on,* viz. *by Tables.*

If a Person has a mind to examine the Divisions on any Line of these Instruments, he will scarcely

scarcely come near his Aim, unless he uses the Line of equal Parts, usually called the Line of Lines, perhaps for its Excellency; together with Tables fitted to that Line you would examine.

But now the principal Difficulty approaches; we are well assured of the necessary Use of such a Line, but how to attain it is the Question.

From the 9th and 10th Propositions of the 6th Book of Euclid, *may be deduced a Method to divide any Line given into any Number of equal Parts, by dividing first another Line into a like Number of Parts; or rather, by taking any Part at pleasure, and laying it as many times on this Line, as the Line given is to be divided into: This is certainly true in Theory, but difficult to be practised, especially when the Divisions are very many, and close together; as will be evident to those who give themselves the Trouble of trying how exactly they can cut 500 Divisions in a Foot.*

To prove whether your Line be cut into equal Parts is easy; for get a sliding Rule of Brass, that will move smoothly, but not too loosely; let the Slider be pinn'd fast to the fix'd Part. Now by the best Method you can think of, divide both Parts together, but let the Divisions be Hair-strokes; which done, then unpin the Slider, and compare the Divisions of it with those of the fix'd Part; thus, move the Slider till the first Division of it falls exactly against the second on the fix'd Part; and then carefully examine whether all the new corresponding Divisions fit and agree. Then slide the same first Division of the Slider to the

 second

second Division of the fix'd Part, and examine as before. Then slide to the third, and so on to the last. Also repeat the same the contrary way, and you will find if your Divisions are exactly equal; and if not, you may estimate how much, and where the Errors are. This to me seems to be an infallible Proof, and a sure Method to correct minute Errors, especially if you use Glasses.

The Line of Equal Parts in this Example was divided into 10 *grand or primary Divisions; each of these were subdivided into* 10 *others of the first inferior Order, or secondary Divisions; and these secondary ones were again subdivided into* 5 *equal Parts, each of which represented two tertiary Divisions, or two of the third Order; those of the Fourth are to be estimated by the Eye.*

And since we were at the Liberty to assign this Line any Length somewhat less than the Leg of the Sector it was taken with, a very great Convenience of such a Length, that of these 500 *Divisions,* $42\frac{2}{3}$ *were equal to one Inch; and the whole Length of the Line was* $11\frac{23}{32}$ *Inches.*

I now proceed to shew how this Line of Equal Parts was made use of in laying down the Sectoral Lines: I mean those Lines which tend to the Center of the Sector, as the Sines, Chords, Tangents, Secants, and Polygons; as also doth this Line of equal Parts.

I say, having transmitted this Line of Equal Parts to the Sector; because every Sine (in *Sherwyn*'s Tables) consisted at most of

of 7 Places, the three last were esteem'd as Decimals, the other as Integers, in order to introduce 4 Places in laying down these Divisions. To do this, because our secondary Divisions were divided but into 5 Parts; the two last Figures were halved, and these Halves used in their stead. Observe, that by the two last, I mean the two last of those I esteem as Integers. Observe farther, that the Divisions we are about to lay down, were taken from the Original Line of equal Parts; and not from a Copy of it.

1. From the Sine of 90, down to the Sine of 5 Degrees, 47 Minutes, all the Sines consisted of 7 Places; and therefore we had four to consider.

Example, The Sine of 41° 15′ is 6593⌞458, which when the two last Figures are halved will be 6546⌞729. Therefore there was taken from the equal Parts 6546, or rather 654$\frac{2}{3}$, because of the Fractions following; *i.e.* there were taken 6 of the principal Divisions, or those of the 1st Order; 5 of the 2d; 4 of the 3d; and $\frac{2}{3}$ of a Distance of a Division of the same 3d Order.

But from 5°⌞47′, down to 0°⌞45′, the Table consists only of 6 Places, and the three last as before are Fractions. But the defective Place in the Beginning was supplied by a Cypher, to compleat the Number of 7. And so every where else.

Suppose we were to lay down the Sine of 3° 30′: I find in the Tables the Sine is 610⌞485, or rather 0610⌞485, and by halving the two last Figures I have 0605⌞242. Therefore I lay down none of the 1st Order;

6 of the Second; none of the Third; and 5 of the Fourth. In like manner the other Sines were laid down.

2. The *Lines of Tangents*, which are in Length, as well as the Sines, equal to the Lines of equal Parts; and so, as was shewn in the third Chapter, would run to 45 Degrees, were laid down in the same manner as the Sines.

3. The *Chords* which run to 60 Degrees, and so (by the 3d Chapter) are equal in length to the preceding Lines, were also laid down by the Table: thus, The Sine of half their Degrees was doubled, and this Double laid down from the equal Parts with the like Management as above.

And each Pair of these Lines makes equal Angles with one another, that they may be used conveniently together at the same opening of the Sector.

4. The Radius of the *Lines of Secants* and of *upper Tangents* of above 45°, were taken exactly equal to the 4th Part of the Length of the Line of Lines.

Therefore in laying them down, we divided the Tabular Secants and Tangents by 4, and then used the Quotients as we did the Tabular Sines.

5. The *Lines of the Polygons* are from a Radius equal to the Length of the Line of Lines. Every Division of them was also taken from the Tables, not only of the Polygons, but of the Chords, Sines, Tangents, and Secants also. Now if the last Place of the Figures be so estimated, that you err not above

above $\frac{1}{10}$ Part of the Diſtance of the laſt Diviſions on the equal Parts; the Error will be leſs than the 400th Part of an Inch in the Line laid down from thence. If the Error in the Eſtimation be not greater than $\frac{2}{10}$ of the laſt Diviſions on the equal Parts, the Error in laying down this Diſtance muſt be leſs than the 200th Part of an Inch. But whoſoever will come and examine the Original, I believe, will be convinc'd that it is eaſy by that Line of equal Parts ſo to eſtimate theſe laſt Diſtances, that the Error ſhall ſcarce exceed $\frac{1}{10}$th Part of one of thoſe Diſtances.

The former 7 Pair of Lines were all that were laid down on the preceding Sector. Of which ſee the Figure.

6. Formerly it was uſual to lay other Sectoral Lines on theſe Inſtruments; *viz. Lines of Superficies, Lines of Solids, Lines of inſcrib'd Bodies, Lines of Metals,* &c. Of which you will meet with a Hint hereafter. Theſe laſt mention'd have been omitted to make room for more uſeful ones, without Confuſion. I deſign alſo a Word or two on Mr. *Samuel Forſter*'s Sector, when the Example in Hand hath been a little ſet-forth.

Beſides the Sectoral Lines (I mean, thoſe beforemention'd which run to the Center) there are ſeveral others running parallel to, and on the outward Edge, whoſe Uſes are very great; and therefore, the laying them down very exactly, requires a great deal of Care and Caution. Theſe moſt properly belong to

the *Gunter*'s Scale; but to render this Instrument compleat are here inscrib'd.

These are, *the double Line of Numbers*, or Scale of *Logarithms*; the *Lines of Logarithmical* or *Artificial Sines*; the *Artificial Tangents*; and the *versed Sines* fitted Back to Back with the right Sines.

Besides a *Line of Inches* subdivided into 8th Parts; a *Line of Inches* subdivided into 10th Parts; a *Line of Foot Measure* into 100th Parts, the *Dialling Lines*, &c.

CHAP. VI.

To Construct the Artificial Lines laid down on the Sector.

FIRST of *the Numbers*: The Sector being quite open'd, and having assign'd one Line to the Right-Hand, perpendicular to the Side of the Sector, and within $\frac{3}{8}$ of an Inch from the End; and having drawn parallel Lines to receive the Divisions: the Length of the forementioned Line of Lines was laid down twice towards the Left-Hand on that design'd for the Line of Numbers, which gave us the Points mark'd 1 at the Left-Hand; 1 in the Middle; and 1 at the Right-Hand; which represent the Places of 1, 10, 100; or 10, 100, 1000; or 100, 1000, 10000.

Then

Then by the Tables you find the Logarithm of 101 to be 2∟0043214; and neglecting the Characteristick, I have ∟0043214, which manag'd as in laying down the Sectoral Sines, becomes first 0043∟214, then 0021∟607. Therefore from the Left-Hand 1, and also from the 1 in the Middle, lay down 0021∟607, or rather 002$\frac{1}{6}$; that is, none of the 1st Order, none of the 2d, 2 of the 3d; and $\frac{1}{6}$ Part of the Distance between those Divisions of the 3d Order, and each of these represent 101.

Again the Logarithm of 102 is 2∟0086002, which by neglecting the Characteristick is 0086002; and then ordering, as in the Sines, you have 0043∟001; that is, none of the 1st Order, none of the 2d, 4 of the 3d, and 3 of the 4th (to be estimated by the Eye) to be laid of the equal Parts from the preceding Points mark'd 1 at the Left-Hand, and 1 in the Middle, which gave us the Divisions for 102.

And the Logarithm of 103 is 2∟0128372; which without the Characteristick is 0128372, and gives 0114∟186; none of the 1st, 1 of the 2d, 1 of the 3d, and 4 of the 4th Order; which laid off on the equal Parts, from the two ones as before, gives the Division for 103.

Thus the Line was continued to the Divisions mark'd 2, denoting 200. But now the Divisions growing closer together, there was not room for every single one; therefore every two only is laid down, *e. g.*

 For

For the Division representing 258: Its Logarithm without the Characteristick is 4116197, which gives 4108∟098; which laid down from 1 gives the Place of 258.

Thus proceed till all the Divisions between the three ones are laid down; diminishing the Number of the Divisions between the Primary ones as they came closer together, as you may see on the Line on the Sector mark'd Number at the End.

2. *The Line of Sines* was laid down from this Consideration; *viz.* That the Logarithmical Co-secant abating the Radius, is the Complement Arithmetical of the Logarithmical Sine to the Logarithmical Radius or Sine of 90°: Therefore the Logarithmical Co-secants, abating the Radius, being laid from the Sine of 90°, which is placed under the End of the Line of Numbers, gives the Sine.

For Example, Let it be required to lay down the Sine of 10°. Its Logarithmical Co-secant is 10∟7603298, and abating the Radius, it is 7603298, and ordering as in the natural Sines, you have 7601∟649; that is, 7 of the 1st Order, 6 of the 2d, none of the 3d, and 1 of the 4th; or rather 7 of the 1st, 6 of the 2d, none of the 3d, but $\frac{1}{6}$ of a Distance of the same third Order; which laid downwards from the Sine of 90° is the Division representing the Sine of 10°.

And for the Sine of 4°, its Logarithmical Co-secant is 11∟1564155, which by abating a Radius becomes 11564155. And by ordering as before; first esteeming the three last Decimals, you have 11564∟155; and

and by halving the two last Figures, it becomes 11532⌞077: That is, 11 of the 1st Order; 5 of the 2d; 3 of the 3d; and 2 of the 4th; which laid down, gives the Division representing the Sine of 4 Degrees.

3. The *versed Sines*, with their Fiducial Line, very near to the Fiducial Line of the right Sines, just now laid down, were constructed from this Consideration; *viz.* That the Distance of the Sine of any Number of Degrees (suppose 40) from the Sine of 90, being doubled, must give us the Division representing twice the Difference (50) of those Degrees from 90 (*viz.* 100) on the Line of the *versed Sines*.

Therefore, half the Degrees of the *versed Sine* to be laid down, and the Logarithm Secant, abating the Radius of those half Degrees, being doubled; and this Double ordered as in the natural Sines, gives the equal Parts to be laid down.

For Example, Let it be required to lay down the Division representing 100°: its half is 50, whose Log-Secant is 10⌞1919325; which abating the Radius and Doubling, is 3838650; and this ordered as in the Sines, gives 3819⌞325; which laid down from the Right towards the Left, gives the Division representing the *versed Sine* of 100.

4. The *Artificial Tangents* run and are number'd the same way with the Sines, till they come to 45 Degrees, which Division falls exactly under the Sine of 90 Degrees: But above 45 Degrees the Divisions are the same with the former, but number'd back-

 wards

wards with ſmaller Figures inverted. So that the Diviſion which repreſents 40 Degrees, alſo repreſents 50; that which repreſents 30, alſo repreſents 60, &c. only the 1 runs upwards as the Sines do, and the other the contrary way.

Therefore the eaſieſt way will be to lay the upper Tangents from 45 downwards, and that will be the Line required.

Thus take any Degrees above 45, ſeek its Logarithmical Tangent in the Tables; neglect a Radius, then order as in the natural Sines.

So for the Tangent of 50 Degrees, whoſe Logarithm is 10└0761865, which by abating the Radius is 0761865, and ordering as in the natural Sines, you have 0730└932; or rather 0731, to be taken off the equal Parts, and from 45° downwards, and it gives the Diviſion repreſenting 50 Degrees which alſo repreſents 40.

I ſhould now proceed to ſhew how the other Lines on the Sector were conſtructed: but becauſe they may elegantly be laid down by the Sectoral Lines already laid down, I omit doing it in this Place, in order to make it one of the Uſes of the Lines already laid down.

CHAP.

CHAP. VII.

To Read, Estimate, and Value the several Divisions and Distances on the Scales or Lines already constructed.

THE Divisions on some of these Lines are determined by the Figures adjacent to them; and these are all of the first Order, and proceed by Tens, and are accordingly numbered by 10, 20, 30, 40, 50, &c. Such are those of *the Chords*, *Sectoral Sines*, *Tangents*, and *Secants*, and also the *Artificial Sines*, *Tangents*, and *versed Sines*.

The Divisions on the *Line of equal Parts*, and on the *Line of Numbers*, which are distinguish'd by 1, 2, 3, 4, 5, 6, &c. are intirely arbitrary, and may signify what Value you please to give them.

I. In the *Sectoral Tangents*, running to 45 Degrees between the numbered Divisions which are of the first Order; they are 9 longer than those adjacent to them, and consequently they form 10 Distances between every two Divisions of the first Order; and each of these denote whole Degrees, every 5th Degree is represented by a Stroke a little longer

longer than the other whole Degrees are repreſented by.

The ſhort Diviſions between thoſe of the ſecond Order, and which therefore are of the Third, are valued by their Number between thoſe of the ſecond Order. Thus, if there be only one Diviſion between the Diviſions repreſenting Degrees, and conſequently two Diſtances; that Diviſion anſwers to the Middle of the Degrees, and then each Diſtance anſwers to 30 Minutes. If there had been two Diviſions, and ſo 3 Diſtances; each Diſtance would have been $\frac{1}{3}$ Part of a Degree, or 20 Minutes.

If 3 Diviſions, and ſo 4 Diſtances; each would repreſent $\frac{1}{4}$ of a Degree, or 15 Minutes. And in this laſt Manner are the Tangents divided in this our Example. But if the Number of the Diſtances had been

$$\left\{\begin{matrix}5\\6\\10\\12\end{matrix}\right\}\text{each would repreſent}\left\{\begin{matrix}12\\10\\6\\5\end{matrix}\right\}\text{Minutes.}$$

If the Compaſs-Point falls between theſe laſt Diviſions, or if you want to take ſome Number of Minutes which does not exactly fall on any one of theſe Diviſions; the Diſtance of thoſe Diviſions muſt be ſuppoſed to be divided into thoſe Minutes they repreſent.

So if there be wanted the Place which repreſents the Tangent of $32^{\circ}. 35'$; and there be three Diviſions, or 4 Diſtances in the third Order between thoſe of the Second; then each

each of these last Distances will represent 15 Minutes, and two of them 30′. But because I want 5 more, I estimate by my Eye a third of this Distance of 15. Therefore from 30 towards 40 I count 2 long Divisions which answer to my 32 Degrees; then I count forward 2 short Divisions, and $\frac{1}{3}$ Part of the next Distance, which Place will represent the Tangent of 32° 35′, as required.

This Explanation of the Tangents will be nearly sufficient for the other Sectoral Lines.

II. You are only to observe, That on the Sines, from 80 to 85, the whole Degrees are only mark'd; and this 85 is a long Stroak as the other Fives are. But from 85 to 90 there was not room for the whole Degrees; therefore from 80 to 85, you must by the Eye estimate the half Degrees; and from 85 the whole Degrees must be so estimated. The Law which these Divisions on the Sines near 90 observe, is this, Their Distances from the Sine of 90 are proportional to the Squares of the Degrees they want of 90; *e. g.* the Distance of the Sine of 85 from 90, is to the Distance of the Sine of 87 $\frac{1}{2}$ from 90, as the Square of 5 (25) to the Square of 2 $\frac{1}{2}$ (6 $\frac{1}{4}$) that is, as 4 to 1. And 85 is distant from 90, 25 times as far as the Sine of 89 is from 90. Hence it appears, That if you will imagine the Distance between the Sines of 85 and 90, divided into 25 equal Parts; of these Parts, 86°, 87°, 88°, 89°, will be 16, 9, 4, 1, distant from the Sine of 90 Degrees.

III. *The*

III. *The Secants* have no Divisions till after the Radial Point, which is at a Hole punch'd against 25 on the Line of Lines. The first Division next after this punch'd Hole is at 10 Degrees; for the Distance is so small, that there is not room for any other Divisions. Between 10 and 20 every two Degrees is laid down; there are 4 Divisions, and 5 Distances, and the Middle of the Distances denotes the odd Degrees. Between 20 and 30, and 30 and 40, every whole Degree is laid down; and there you may judge the Halves by the Eye. Between 40 and 50, the Distances of the Degrees are greater, and the half Degree is laid down. From 50 to the End, which is at about 76, every Degree is divided into Quarters by 3 Divisions of the 3d Order, like the Chords and Tangents; and therefore to be valued like them.

IV. Of the *Artificial right Sines*, between 80 and 90 there is but one Division, and it denotes 85. The Law to be observed in the Estimation of the other intermediate Degrees is nearly the same with that laid down for the natural Sines among the Sectorals. From 80 to 60, every whole Degree. From 60 to 30, every half Degree. From 30 to 20, every Quarter; and from 20 to 10, every 6th Part of a Degree; that is, every 10th Minute. And from 10 down to the End of the Scale, every Degree is divided into 12 Distances; and so every Distance denotes 5 Minutes.

5 Minutes. All these intermediate Spaces are to be subdivided by the Eye; and in this Part of the Line you may estimate the $\frac{1}{2}$ of a Minute.

V. Of the *versed Sines* running Back to Back with the Sines, but counted the contrary way; the first 10 Degrees have no Divisions, nor can there be any distinct, for the versed Sine of 5 Degrees, is but $\frac{1}{4}$ of the versed Sine of 10 Degrees. From 10 to 20 every second Degree is laid down, there being not room enough to express any more distinctly. From 20 to 90, every whole Degree. From 90 to 130, every half Degree. From 130 quite to the End, to every Quarter. In this Part of the Line you may judge very well to 2 or 3 Minutes.

VI. The *Artificial Tangents* have the Divisions of the 1st Order number'd 10, 20, 30, 40, 45; which 45 comes to the End under the Sine of 90: And on the same Divisions it is numbered backwards by smaller Figures inverted, from 45 to 50, 60, 70, 80, and continues on with Divisions of the 2d Order, to 81, 82, 83, 84, 85, 86, 87, 88, 89, as far beyond as to about 89°. 20′. Sometimes every one of these Divisions are numbered by a small Figure, and sometimes only the 85.

From the Beginning 50′ upwards to 10, and from 80 downwards to 89°. 20′, every Degree is divided into 12 Parts, that is, into every 5th Minute. From 10 to 20, and from

I

from 70 to 80; into 6 Parts, and so every Distance denotes 10 Minutes. In the two preceding Ranges of Divisions you will scarce err above one Minute. From 20 to 45, and back again to 70, the Degrees are every where divided into 4 Parts; and so each is 15 Minutes: And here you will scarce err above 2 or 3 Minutes.

But you may observe, That the Artificial Sines, like the Tangents, are number'd backwards, with the Figures inverted; thus the 10 falls on the Division of 80; the 20 on that of 70; the 30 on that of 60, &c. And this by the Construction of the Sines is an Artificial or Logarithmical Line of Secants, and so entitled in a small Letter engraven in an inverted Position.

VII. The *Sectoral Line of Polygons* needs no Explanation, for the punch'd Holes are number'd with the Figures 6, 7, 8, 9, 10, 11, 12, denoting the Number of the Sides of that Polygon. And this admits of no Sub-divisions.

We said the Line of Lines, and the Line of Numbers had their Divisions arbitrary.

VIII. *The Line of Lines* is divided into 10 equal Parts, numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and these I call Divisions of the first Order. Each of these are again sub-divided into 10 other equal Parts, which are of the second Order; each of these last are subdivided into 5 equal Parts, which by estimating the Middles of the Distances may be

be ſaid to be into ten, which are of the third Order. And ſo the whole Line would be divided into 1000 equal Parts.

Now if the whole Line be conceived to be divided into 1000 Wholes or Integers (which in Practice it very well may, without the leaſt Confuſion); the Diviſions of the firſt Order, that is the numbered ones, will repreſent Hundreds; ſo the Figures 1, 2, 3, *&c.* denote 100, 200, 300, *&c.*; the Lines of the ſecond Order muſt denote Tens, *viz.* 10, 20, 30, *&c.* And thoſe of the third Order Units only.

When the whole Line repreſents 100; then the Diviſions of the firſt Order will be Tens; and ſo 1, 2, 3, *&c.* will repreſent 10, 20, 30, *&c.* Thoſe of the ſecond Order will be Units; and thoſe of the Third will be tenth Parts.

If the whole Line repreſents 10, the Diviſions of the firſt Order will repreſent what their Numbers annexed ſhew; thoſe of the Second, Decimal Parts; and thoſe of the Third, Centeſimal ones.

Laſtly, When the whole Line muſt repreſent 10000 Wholes or Integers; the Diviſions of the firſt Order denote Thouſands: ſo the Figures 1, 2, 3, *&c.* denote 1000, 2000, 3000, *&c.* The Diviſions of the ſecond Order, will denote Hundreds, thoſe of the Third, Tens. And if the half of one of theſe ſmall Diſtances be conceived divided into ten Parts, thoſe Parts will be Units.

Moreover,

Moreover, whatsoever Value you affix to the Divisions on one Leg of the Sector, the same Value must be given on the like Divisions on the other Leg.

IX. In the two Lines of Numbers the one beginning where the other ends, the Divisions of the first Order are number'd thus, 1, 2, 3, 4, 5, 6, 7, 8, 9; 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Between every one of these Divisions of the first Order, there are 9 Divisions of the Second, and so 10 Distances. The Divisions of the third Order are alike in both Lines. From 1 to 2 there are 10 Distances of the third Order between every two Divisions of the second Order. From 2 to 3 there are but 4 Divisions, and so 5 Distances, the Middles of these Distances being estimated, you will have all the 9 Divisions of the third Order. From 3 to 10 the Distances of the second Order are only cut into two Parts, each denoting 5 Distances of the third Order; the other Distances of this third Order are to be estimated by the Eye, and to be used with very sharp-pointed Compasses.

When the Divisions in the first Line denote Units, those in the second denote Tens; when those in the first are Tens, those in the second are Hundreds; when Hundreds, those in the second will be Thousands. And the Value of the Divisions of the second Order are regulated by those of the first Order, according to the Rules laid down for the Line of Lines. That was, if those of the first Order were Tens, those of the second

were

were Units, and those of the third, Decimals. If those of the first were Hundreds, those of the second were Tens, and those of the third Units, &c.

Lastly, though the Examples here taken relate only to the Foot Sector before mentioned (knowing that if the Sector had been longer, more Divisions might have come on, but if shorter not so many) yet the general Rules herein deliver'd duly observed, are sufficient to shew the Use of all Sizes of Sectors.

CHAP. VIII.

Of the Sector *in general, with general Laws for working by the* Sectoral Lines.

IN respect of its Use, it is applicable to every Thing wherein Proportions are made use of.

The Ground of the Sectoral Lines depends on the 2d and 4th Propositions of the 6th Book of *Euclid's* Elements.

The Ground of the *Artificial Lines* depends on the Nature and Properties of Logarithms.

In the Use of the *Sectoral Lines*, two Openings of the Compasses are always necessary.

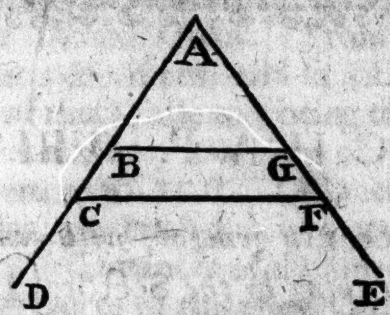
In the *Artificial Lines* seldom above one.

For where there is but one Proportion, and that wrought by one or two Lines at moſt; this is the general Law.

Extend the Compaſſes from the firſt Term to one of the Middle ones; and the ſame Extent applied the ſame way, will reach from the other of the Middle ones, to the Anſwer.

In this Chapter, I ſhall only conſider the *Natural* or *Sectoral Lines*, being thoſe that meet together at the Center. And ſhall hereafter in a proper Place, particularly treat of the *Artificial* ones.

All the Lines as AD, AE, meeting



in the Center, are of an equal Length; and thoſe that bear the ſame Name are equally and alike divided and number'd. And therefore in all the Operations wrought by the *Sectorals*, the Points B, G, repreſenting the ſame Diviſions, are equally diſtant from the Center A. The ſame is true of any other two Points, as C and F. And therefore, AB is to AC, as BG is to CF, by the 2d and 4th Propoſitions of *Euclid*'s 6th Book.

N.B. The

N. B. The Lengths AB, BC, as alſo AG, GF, are call'd *Crurals*, becauſe they lie, or are meaſur'd in the Legs of the Sectors.

The Lines BG, CF are call'd *Parallels*; becauſe in all Proportions work'd by the *Sectorals*, the two Extents between the Legs will be two ſuch Lines parallel.

If AC on the *Line of Lines*, meaſures 67 Parts, and ſo AF likewiſe; then is AC and AF ſaid to be 67 Crural Parts, and CF 67 Parallel Parts.

If AC were the Chord of 39 Degrees; then is AC the Crural Chord, and CF the Parallel Chord of 39°.

In working Proportions by theſe Lines, the firſt Term may in moſt Caſes be Crural or Parallel. When the firſt is $\left\{\begin{matrix}\text{Crural}\\ \text{Parallel}\end{matrix}\right\}$ the laſt or ſought Term is $\left\{\begin{matrix}\text{Parallel}\\ \text{Crural}\end{matrix}\right\}$ that is the laſt is in a Poſition different from the firſt.

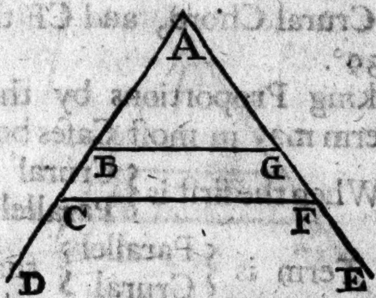
Of the three Terms given, to find the Fourth; the firſt Term, and that which is of the ſame Denomination, whether it be the 2d or 3d, I call Terms of the firſt Denomination: And the other Term, with that ſought, I call Terms of the ſecond Denomination.

N.B. To the *Line of Lines* run three others parallel, to receive the Diviſions and Numbers. But that is the Line to be uſed which receives all the Diviſions; it is that next the inner Edge of the Sector, and it runs exactly into the Center; whereas the others paſs by,

or ſtop before they come at the Center. The ſame is to be underſtood of all the other Sectoral Lines.

The general Laws for working Proportions by the Sectoral Lines are theſe three following; according as you would have the firſt Term Crural or Parallel.

Firſt, When the firſt Term is to be Crural. Let the three given Quantities be R (50),



S (60), T (70), to find a fourth Proportional; and let R and T be Feet; and 60 and the Quantity ſought be Shillings. Find on both the Legs AD, AE, the Lengths of the Terms of the firſt Denomination, all counted from the Center; that is, take AB, AG, each equal to R (50), and AC, AF, each equal to T (70). Then open the Sector till S (60) of the ſecond Denomination, reaches from 50 to 50, the Ends of the firſt Term; ſo ſhall the Diſtance from 70 to 70 be (94); the Anſwer, if meaſur'd on the ſame Line the other was meaſur'd on.

Secondly, When the firſt Term is to be parallel, take (50) the firſt Term in the Compaſſes, and open the Sector till it reaches (from 60 to 60) the Term given of the 2d Denomination. The Sector being thus open'd, take (70) the other given Term in your Compaſſes, and draw it along the Legs of the Sector till the Points reſt on the ſame Diviſions of both Legs; and thoſe Diviſions ſhew the Anſwer or Quantity ſought (in this Example 94) as before.

Here it may be obſerv'd, that the Operation by the ſecond Rule is perform'd by Tentation; in the firſt without.

The ſecond Rule ought to be avoided when the Sector is to be open'd to a very ſmall Diſtance.

Thirdly, If the firſt Term of the Proportion be made Crural, and the Diſtance of the Points repreſenting the firſt Term, when it is the greateſt that the Sector will admit of, is ſhorter than the given Term of the ſecond Denomination; either the Term of the ſecond Denomination muſt be made Crural, or taken from a ſmaller Scale, or ſome of the Variations hereafter laid down muſt be made.

When the 4 Terms of the Proportion are all of the ſame Denomination; either the ſecond or the third may be compar'd with the firſt.

The five following Variations may be allow'd when Conveniency requires it.

 1. The

1. The first Term, and either of the middle Terms may be multiply'd or divided by any one and the same Number.

2. If you multiply or divide the first Term by any Number, and work with the Result; you must accordingly multiply or divide what arises from that Work by the same Number, and you will have the Solution.

3. You may $\left\{\begin{matrix}\text{multiply}\\ \text{divide}\end{matrix}\right\}$ either of the middle Terms by any Number, if you will $\left\{\begin{matrix}\text{divide}\\ \text{multiply}\end{matrix}\right\}$ the other by the same Number.

4. If you $\left\{\begin{matrix}\text{multiply}\\ \text{divide}\end{matrix}\right\}$ either of the middle Terms by any Number, and work with the Result; you must $\left\{\begin{matrix}\text{divide}\\ \text{multiply}\end{matrix}\right\}$ what arises from that Work by the same Number, and you will have the Solution.

NB 5. The Parallels may be taken from any one Scale whatsoever, as well as from those Scales laid down on the Sector.

N. B. Of Compasses the Beam Compasses are fittest to be used with the Sector; because their Points are always perpendicular to the Plane of the Sector.

When the Sector is to be open'd by a Crural Term which falls near to the Center; multiply that Crural Term and its correspondent Parallel, till they produce Numbers at a sufficient Distance from the Center. The farther the better.

CHAP.

CHAP. IX.

Of the Use of the Line of Lines.

IT is impoſſible to write all the Uſes to which this Line is applicable. It is ſufficient in this ſmall Tract, to lay down ſome that are eaſy, and fit for the common Practices of Life. Such as are every Day required by the Architect, the Working Builder, the Land Surveyor, the Watch Maker, the Mathematical Inſtrument Maker, *&c.*

I. To make a Scale of a deſired Length, that ſo a Draught deſign'd to be laid down by it, may come within an aſſign'd Compaſs.

Example. To make a Scale that may repreſent 100 Yards in 1 Foot; or, which is the ſame Thing, to divide a Foot into equal 100 Parts.

Take in your Compaſſes (1 Foot) the Length of the Scale, and open the Sector till the Compaſs-Points reach (100) the Parts to be contain'd on that Scale, on both Legs. The Sector thus open'd, Take the Diſtance from 90 to 90 in your Compaſſes, and it will be 90 Yards on your Scale. And the Diſtance from 57 to 57 is 57 Yards on the Scale. So every Diviſion may be thus transferr'd to your Scale, or the Sector thus open'd is your Scale.

That you may not be at a Loſs in underſtanding other Treatiſes on the Sector, and perhaps, ſome of the following Paſſages in this; obſerve the Stile in the following Words containing the preceding Example.

Make 1 Foot the Parallel of 100; and the Parallels of 90 and 57, are 90 and 57 Yards, *&c.*

Example II. To divide a Foot into 96 equal Parts.

Take 1 Foot in your Compaſſes, and open the Sector till the Compaſs-Points reach from 96 to 96; then the Diſtance from 50 to 50 gives 50 of thoſe Parts required.

In the Sectoral Language, make 1 Foot the Parallel at 96, and the Parallel at 50 is 50 Parts.

Example III. To divide 1 Foot into 120 equal Parts.

Here it is to be obſerv'd, that if the Diviſions of the firſt Order be made Tens; 120 will be beyond the End of the *Line of Lines*: And if I make the ſame Diviſion Hundreds, the 120 cannot be open'd at the Diſtance of a Foot, which is the Line to be divided.

Therefore divide or multiply the Foot, and alſo the 120 Parts by any Number which will ſuit you beſt. Here in this Example, if you divide by 2, the Quotients are $\frac{5}{10}$, or half of a Foot, and 60 the Parts it is to be divided into. Therefore by the preceding Doctrine, divide half a Foot into 60 Parts, and it is the Scale requir'd. Thus, open the Sector till

half

half a Foot reaches from 60 to 60; and your Sector as far as 100 is the Scale you want.

Hence appears one Beauty of this *Line of Lines*; *viz.* that it is a Scale of equal Parts of all Lengths: And is therefore preferable to any Scale where the Divisions are only of one particular Length.

Example IV. To form a Scale of 1 Foot, 9 Inches, that may represent 145 Yards.

Here I cannot open the Sector wide enough to receive 1 Foot, 9 Inches; therefore I multiply both by some Number, suppose 4, and there will arise 7 Foot, and 580 Yards; then I divide all by 7, and find 1 Foot, and 82|85 Yards. Therefore opening the Sector till 1 Foot reaches from 82|85 to 82|85, it will be a Scale of 100 Yards of the Bigness requir'd.

On or near the outer Edges of the Sectors are usually plac'd 3 Lines of equal Parts, *viz.* a Foot Line in 100 equal Parts. The Manner of laying this down is in the first Example.

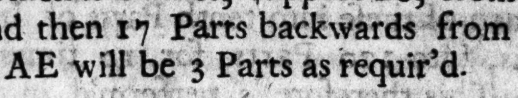
The 2d is a Foot Line divided into Inches, and each of these into 8 Parts. This is constructed in the second Example, for (12 times 8 is 96) and there the Foot was divided into 96 equal Parts.

The 3d is a Foot Line divided into Inches, and each of these into tenth Parts of Inches; that is, into 120 Parts, for 10 times 12 is 120. This therefore is perform'd in the 3d Example.

And now I have in some measure, or partly perform'd the Promise I made in the last

last Paragraph of the 6th Chapter. These Lines are also mention'd, and a little describ'd in the last Paragraph of the 5th Chapter.

V. When you are about to take from the Sector those Divisions which are near the Center, in order to transfer them to the Scale you are making; or if you use the Sector open'd properly for your Scale, and you are to lay down a small Number of Divisions, suppose 3, you may be a little confus'd. For there are two Lines which run to the Center on each Leg; and to avoid Confusion, as much as may be, the Lines parallel to the Fiducial ones, are cut off (from one Line on each Leg) two Inches from the Center, and the Fiducial Lines only continu'd, with prick'd Holes instead of Divisions; and yet even then, very near the Center, the Distances are scarce distinguishable. Observe the Remedy, If you are to lay from



A towards D, 3 Parts; lay any larger Number, which is distinct, suppose 20, from A to C; and then 17 Parts backwards from C to E; so AE will be 3 Parts as requir'd.

VI. If a Draught of Land, Buildings, &c. be drawn without a Scale, and you would add a Scale to it:

Measure

Measure on the Land or Building any one Line not the shortest, suppose it to be 50 Poles $\frac{1}{2}$, or Feet. And let that Line be represented in the Draught by HI.

Take the Line HI in your Compasses, and open the Sector till the Points reach from 50 $\frac{1}{2}$ to 50 $\frac{1}{2}$, the Measure of HI; and so will the Sector thus open'd, be a Scale of 100 Poles or Feet for that Draught, to which it may be transferr'd.

VII. To increase or diminish a Line in any Proportion assign'd.

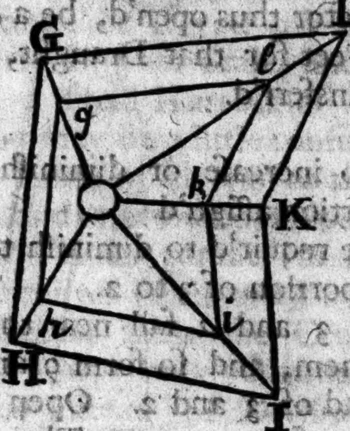
Let it be requir'd to diminish the Line HI in the Proportion of 3 to 2.

Because 3 and 2 fall near the Center, I multiply them, and so form 9 and 6, which I use instead of 3 and 2. Open the Sector till 6 reaches from 9 to 9. Then laying the given Line AB from the Center, keep the upper Leg of the Compasses in the Place to which it reaches, and turn the other over to the same Place in the other Leg; and the Distance now in the Compasses is the Line diminish'd as requir'd.

VIII. In like manner may the other Sides, and also the Diagonals be diminish'd, even without altering the Sector. For take IK in the Compasses, and lay it from the Center on the *Line of Lines*; and keep fast the upper Point, turn the other about till it falls on the same Part on the other Leg; and the Distance of the Compasses will give BC diminish'd. Hence any Figure may be diminish'd at pleasure,

pleasure. But a Figure of many Sides may be reduc'd shorter thus:

IX. Figure 7. Numb. 1. is to be reduc'd to a less in a given Proportion.



Any where in the Figure take a Point O; draw Lines OG, OH, OI, OK, OL, to all the Angles. Diminish these Lines in the Proportion assign'd, and let them be Og, Oh, Oi, &c. and draw *gh*, *hi*, *ik*, &c. and the Figure is diminish'd as requir'd.

X. If only one of these Lines had been diminish'd, suppose OG to Og; and then the Lines *gh*, *hi*, *ik*, &c. had been drawn parallel to GH, HI, IK, &c. the Figure would have been diminish'd as before. But the Method laid down in the 8th Article is the better way; because the Points g, *h*, *i*, &c. are found independant on any other. But in

in the latter, the Points *i*, *k*, *l*, &c. depend on the preceding ones.

XI. The same had been equally true, if the Point O had been taken without the Figure, or on one of its Sides, or in one of its Angles. (Fig. 7. Numb. 2, 3, 4.)

Fig. 7. Numb. 2.

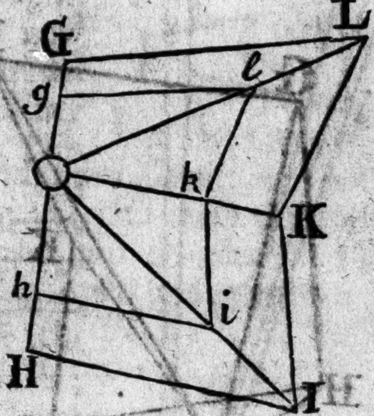
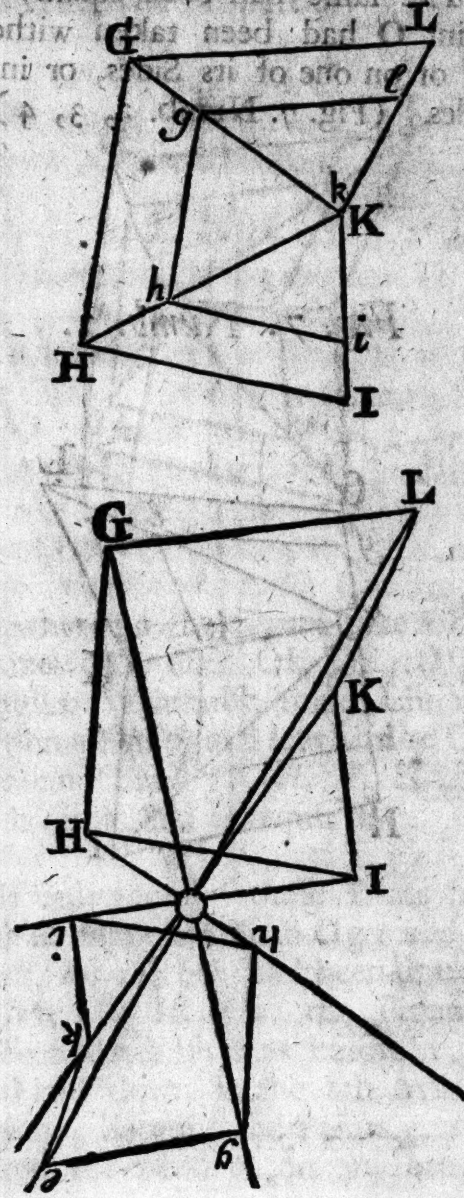
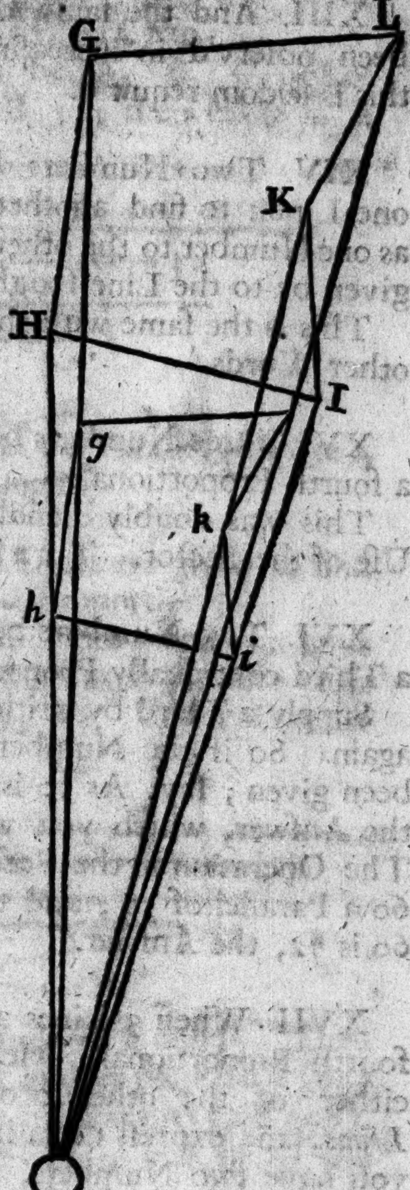


Fig. 7. *Numb.* 3, 4.



XII. The

XII. The ſame had been alſo true, if the Point O had been taken without, and the diminiſh'd Lines had been laid towards the contrary Parts from O, as in Fig. 7. N°. 5. And this is a very excellent Method, becauſe you do not offend the 1ſt Draught. And the new Draught is drawn clean without any transferring. This Problem, thus manag'd, deſerves a much larger Explanation, and to be exemplify'd in many Varieties; but here I have not Room. I have fully ſhewn the Uſe of the Sector herein; and therefore leave it for another Place.



Only obſerve that there are particular Inſtruments for this purpoſe.

XIII. And the ſame Method might have been obſerv'd in increaſing a Figure; but this is ſeldom requir'd.

XIV. Two Numbers being given, and one Line: to find another Line ſuch; that, as one Number to the other, ſo may the Line given be to the Line ſought.

This is the ſame with the foregoing, but in other Words.

XV. Three Numbers being given; to find a fourth Proportional.

This was doubly handled in the general Uſe of the Sector. Turn back to Chap. 8.

XVI. Two Numbers being given; to find a Third continually Proportional.

Supply a Third by writing the Second over again. So if the Numbers 50 and 60 had been given; ſay, As 50 is to 60, ſo is 60 to the Anſwer, which you will find to be 72. The Operation in the Sectoral Stile is, make 60 a Parallel of 50; and then the Parallel of 60 is 72, the Anſwer.

XVII. When 3 Lines are given, to find a fourth Proportional. Meaſure the firſt and either of the other two on the *Line of Lines*, and expreſs them in Numbers; then you have two Numbers, and a Line given: to find another Line to which that given bears ſuch Proportion, as the Number expreſſing the Meaſure of the firſt Line given, doth

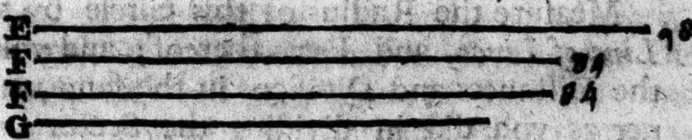
doth to the Number expressing the Measure of the other measur'd Line. And this is the same with Art. 13th of this Chap.

Let the 3 Lines be A, B, C. Let A measure 70, and B 50; then make C a

Parallel to A (70), and the Parallel of B (50), will be D the Line sought.

XVIII. When two Lines are given, to find a Third in a continual Proportion;

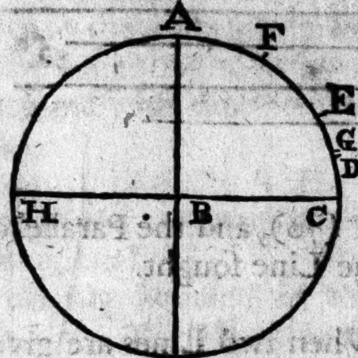
Consider the Second twice; and you have 3 Lines given, then work as in the last. Let the two Lines be E, F. Under F draw



another equal to it, and call it also F. Then measure the two first, and let them be E (98) F (84). Then make F a Parallel to E (98), and the Parallel of F (84), will be G the Line sought.

XIX. To divide a large Circle, Quadrant, Sextant, Dodrant, &c. into Degrees.

First the Circle is eaſily quarter'd, by raiſing BA perpendicular to the Diameter CH; then Sextants and Dodrants are



had, by laying the Radius from A to E, and from C to F; or by biſſecting the Arch CF. Biſſect the Arch CE in D, and the Arches ED, DC, will be each of 15 Degrees.

Meaſure the Radius of this Circle by the *Line of Lines*, and Parts thereof; and value the Diſtances and Diviſions in the ſame manner as you did in dividing the Sector; ſo you will have four Places in the Decimals, if you want them. Let this Meaſure be (for an Example) 5 times the Length of the *Line of Lines*, and the following Parts; *viz.* one of the 1ſt Order; 4 of the 2d; and 2 of the 3d; ſo conſider'd as the 2d, are divided into 5 Parts; and then Decimally, calling the whole *Line of Lines* one Unite, the Length is 5⌊144.

Then

Then from the Tables of Sines calculate the Sine of 30 Minutes, and double it; so the Chord of 1 Degree will be └089778, where the whole Line is the Integer. But when the Divisions of the 1st Order are Integers, the Chord of 1 Degree to that Radius is └89778. And this according to the Method laid down in using the Sines, is 8938└9; which therefore may be called 8939, for it differs from it but $\frac{1}{50000}$ Part of an Inch.

Therefore open the Sector till 1 of the 1st Order will reach from 10 to 10. And then take the Parallel of 8939; and it shall be the Chord of one Degree answerable to the given Radius. And taking this Distance with hair Compasses, and using a Glass, you may be very exact; for the Sector thus open'd is a Diagonal Scale of 50000 in the Foot, supposing each of the small Distances divided into 10 equal Parts by the Eye. Now as oft as you err $\frac{1}{3}$ of these Distances, so oft will you err 2 Seconds of a Minute, and no more.

Lay this Chord from D to G; and the Arch GC will consist of 16 Degrees; and consequently by a continual Bissection may be divided into single Degrees.

By repeating this Bissection you may have the Halves and Quarters.

If from the Arch of 45° be taken the Arch 2°└20′, or from the Arch of 22°└30′ be taken the Arch of 1° 10′, &c. by a continued Bissection you may attain every 10th Minute; and so consequently every fifth.

 If

If to the Arch of 7° 10′ be added the Arch of 62 Minutes, or to the Arch 3° 45′ be added the Arch of 31 Minutes; the Arch of every single Minute may be had by Bissection.

Dr. *Halley* so well approv'd of Divisions by Bissection, that he caused his large mural Quadrant of 8 Feet Radius, to be divided into 96 equal Parts, because he could attain these Divisions by a continual Bissection of the Dodrantal Arches. And to reduce these to the Degrees, hath a Table calculated, whereby they appear, as it were, by Inspection.

CHAP. X.

Of the Use of the Lines of Chords *and* Polygons.

THE Chord in this Example runs only to 60 Degrees, yet is sufficient to lay down any Angle, or measure one, tho' never so great, and more exactly than can be done by those which run to 90 or 180 Degrees.

By those you must open the Compasses twice, by this you do no more; by those you cut the Arch obliquely (which is a great Dis-

Disadvantage) by this your Compass Points never make with the Arch an Angle less than 60 Degrees.

Besides, if any one affects these Lines of Chords to many Degrees, they will find them supplied in the Use of Sines, even to the 180 Degrees; tho' for my Part, I am not fond of Chords much above 90°.

I. The Radius AB and Degrees

of an Arch, suppose 56, being given; to find the Chord CD.

Take the Radius AB given, and open the Sector till the Compasses reach from 60 to 60. Then the Distance from 56 to 56 is the Chord required.

II. The Chord CD, and the Degrees 56 being given; to find the Radius.

Open the Sector till the Chord CD reach from 56 to 56, and then the Distance from 60 to 60, is AB the Radius.

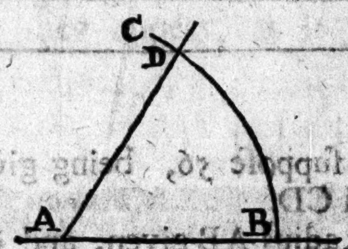
III. The Radius AB, and Chord CD being given; to find the Degrees.

Make AB a Parallel Radius, that is, open the Sector till it reaches from 60 to 60. Carry the Chord CD along the Legs till they rest on

like Divisions in the Fiducial Lines, which you will find to be at 56°.

The principal Use of the Sector is to make Angles of assign'd Magnitudes, with given Lines, and at a given Point thereof. And also to measure those Angles that are made, &c.

IV. Let there be a Line AB and at A, a given Point thereof, let it be required



to make an Angle (of 51° 30′) less than of 60 Degrees.

This may be done two different Ways without opening the Sector. By taking the whole Radius of the Line of Chords to describe the Arch with, or by taking the Distances of the 60's to describe the Arch with. And in the former Case to take the Crural Chord of 51° 30′; and in the latter the Distance of 51° 30′ to 51° 30′. Which will be easily understood after the Solution of this first Example, universally.

Open the Sector at Pleasure, and in the Compasses take the Distance from 60 to 60; and

and setting one Foot in A, with the other describe the Arch BC; the Sector remaining thus open, take the Distance from 51°∟30′ to 51°∟30′; and lay it on the Arch from B to D; then draw AD, and the Angle DAB is of 51° 30′, as requir'd.

If any one Radius would have been more convenient than another; take that in your Compasses, and describe your Arch, and open the Sector till that Radius reaches from 60 to 60; then as before, take the Distance from 51∟30 to 51∟30, and lay it off upon the Arch, and draw AD; it is done.

This in the Sectoral Stile is thus; make the convenient Radius a Parallel at the Chord of 60; and then the Parallel at 51°∟30′ is to be laid on the Arch.

Or thus; Make the convenient Radius a parallel Radius; and the parallel Chord of 51∟30 must be laid upon the Arch.

V. If the Angle to be made had been very small (suppose of 3 Degrees) which to make would be troublesome, on account of the approaching of the Lines towards the Center; See Fig. 1. following.

From the given Point E, with a convenient Radius describe the Arch FG; and lay this Radius on the Arch from F to H (which will be 60 Degrees): Then make this Radius a Parallel of 60; so will the Parallel of (57) as much as your small Angle wants of 60, reach backwards on the Arch from H to I. Draw EI, and the Angle at E is of 3 Degrees, as required.

VI. If the Angle to be laid down be of more than 60° (suppose of 94.) See Fig. 2. following First with any Radius describe the Arch LO, from the given Point K; and on it lay the same Distance from L to M; so will LM be 60 Degrees; now make the convenient Radius a Parallel at 60; then take the Parallel at (34), the Degrees which were given above 60, and lay it from M to N, and draw KN; so will the Angle K be of 94 Degrees as required.

VII. If the Angle to be made be of more than 120 Degrees (or more than two 60's) suppose of 167 Degrees; See Fig. 3. following:

With any convenient Radius describe the Arch QU; lay the same on it from Q to R, and from R to S; so will QS be 120 Degrees, too little by 47. Then make the Radius a Parallel at 60; and lay the Parallel at 47 from S to T, and draw TP; then will QT be 167°: An Angle made by TP and PQ, will be an Angle of 167°

VIII. If in the two last Cases the Degrees to be laid off after the 60's, had been very small, they might have been managed as in the 2d Example.

IX. In the fourth Example, Fig. 3.; if the Arch WS had been describ'd, and as many Degrees laid from W to T, as the required Angle wanted of 180; in this Example 13: And then the Line TP be drawn;

drawn; the Angle made by TP, and TQ would be of 167°, as before.

X. Suppose the preceding, or any other Angles were to be measured;

1. With any convenient Radius from the angular Points A, E, K, P, (Fig. 1, 2, 3.) describe Arches meeting the Legs of

Fig. 1.

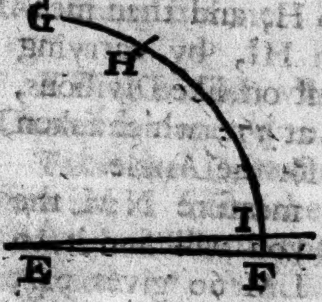


Fig. 2.

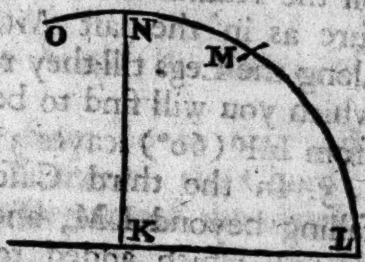
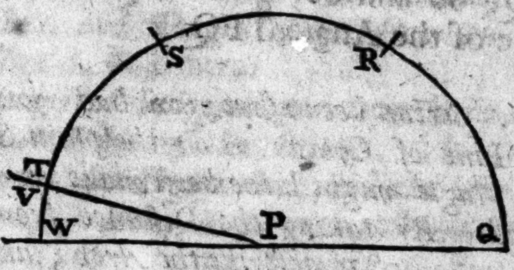


Fig. 3.



the Angles; and where the Arch is greater than 60, or twice 60 (which will be known by applying the Radius to the Arch) turn the Radius over the Arch once, as you will find

find in Fig. 1; and twice, as you will find in Fig. 2. but never a time in the two Figs. p. 53, 54.

2. Now make your convenient Radius a Parallel at 60, and leave the Sector at this Opening.

3. Then in the 1st Example, take BD in the Compasses, and draw them over the Legs till they rest at like Divisions on the Fiducial Lines, and this you will find to be on 51, 30; the Degrees of the Arch DB, and so also of the Angle A.

4. In the Second, because very small, lay off the Radius from F to H, and then measure as in the last Arch HI, by carrying along the Legs till they rest on like Divisions, which you will find to be at 57; which taken from HF (60°) leaves 3° for the Angle.

5. In the third Case measure NM the falling beyond LM, and you will find it to be 34°, which added to LM 60 gives 94°, which is NL the Measure of the Angle K.

6. In like manner TS 47, added to SR and RQ, 60 and 60, gives TQR 167°, the Measure of the Angle TPQ.

Had the Sector been shut, or had you but one Line of Chords as on the plain Scale, any Angle might have been made or measured. By taking the Distance from the Center to 60° to describe the Arch; and the Distance from the Center to the Degrees of the required Angle to lay on the Arch.

But then the Radius is fix'd to one Length; whereas on the Sector it may be any Length between quite shut, and so open'd

open'd as to put the 60's at the greatest Distance from one another; which differs very little from being quite open'd.

Here again it appears how much more the Sector is preferable to all other Instruments for these and the like Purposes.

XI. To open the Sector, so that the Lines of Chords may make any given Angle with one another, not exceeding 60 Degrees.

Take the Distance from the Center to the given Degrees, and make it a Parallel at 60, the Thing is done.

XII. To find what Angle the Lines of Chords are open'd to.

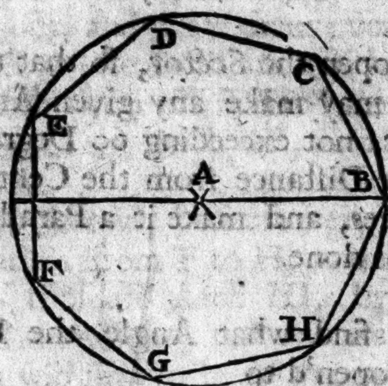
Take the Distance from 60 to 60, and measure it from the Center on the Chords, and you will have the Degrees of the Angle required.

In the Use of the Sines will be shewn how to open the Sector so that Chords, Lines, Sines, and Tangents, may form any Angle assign'd, & contra.

The Lines of Polygons are very near to the inner Edge of the Sector. At the End stands the Figure 6. And the other small punch'd Holes in it are numbered towards the Center by the Figures 7, 8, 9, 10, 11, 12.

XIII. To

XIII. To inſcribe a regular Polygon of 7 or any Number of Sides in a given Circle.



Open the Sector till (AB) the Radius of the Circle reaches from 6 to 6, and then the Diſtance from 7 to 7 will be BC the Side of the Polygon ſought; which will divide the Circumference of the Circle into 7 equal Parts, in the Points B, C, D, E, F, G, H. Draw the Lines BC, CD, DE, &c. and you have the Polygon required.

XIV. And thus you may work for a Polygon of 8, 9, 10, 11 or 12 Sides.

XV. If you want a Polygon of 5 Sides, divide the Circumference into 10 equal Parts, and then every two will be a fifth Part.

XVI. If

XVI. If you want one of 3 Sides, the Radius will divide the Circumference into 6 Parts, and every two of these will be a 3d Part.

XVII. If the Polygon you want consists of more Sides than 12 (as for Example 15); divide 360, the Degrees of the whole Circumference, by (15) the Number of the Polygon's Sides; and to the Radius of the Circle in which the Polygon is to be inscrib'd, find the Chord of (24°) the Quotient: And it is the Side of the Polygon sought.

XVIII. But if it had been required to divide the Circle into many equal Parts (suppose 32) the Work is best performed by taking first the Chord of (11° 15′) one of those Parts; then of (22, 30) two of those Parts; then of (33, 45) 3 Parts; then of (45) 4 Parts; of (56, 15) 5 Parts; of (67, 30) 6 Parts; of (78∟45) 7 Parts; of (90) 8 Parts: And then from one and the same Point, suppose C, lay on the Arch the Chord of one Part; then from C lay the Chord of 2 Parts; from C lay the Chord of 3 Parts, &c. till you arrive at or near the Chord of 90 Degrees; call this last D. Now begin from D, and work as before you did from C, proceeding onwards till you come to, or near to 90 Degrees. And then leave off and begin again.

The Reason of this Process will easily appear. For if you had taken the Chord of the

32d

†

32d Part of the Circumference, so nearly true, that a sensible Error was not found; yet by turning the Compasses over 32 times, it's very likely some Error would have appeared at last.

But by taking 6, 7, 8, or more of these Parts before I begin again, I have no Dependance on, and therefore do not communicate the Errors of the intermediate Divisions; which Errors tho', singly consider'd, are very minute, yet when multiply'd by 32, produce one very sensible.

It is the best way to begin again when you have about 90 Degrees; because afterwards should you go farther, the Arches struck with the Compasses would cut the Circumference of the Circle too obliquely.

If a Circumference were so divided, and Lines drawn from those Divisions to the Center, it would represent the Mariners Compass. And if this be laid down in a Sea-Chart; and these Lines continued every way to the utmost Extent of the Chart, they would be the Rhumbs.

In dividing the Periphery of a Circle into many equal Parts (suppose 45) you will no ways fall exactly on the 90, but frequently fall short of it, or exceed it, as in the 45; for 11 Parts would be short of the 90, and 12 would exceed it. Therefore in this Case it is best to renew the Work after you have laid down the 11.

XIX. If the Side and Number of Sides of any Polygon be given; the Radius of the

the Circle in which it may be inſcrib'd, may be found.

Let the Polygons Side be BC in the Fig. aforegoing, and the Number of Sides be 7.

Open the Sector till (BC) the Side given, reaches on the Polygons from (7 to 7) the Numbers of the Sides; then will the Diſtance from 6 to 6, be (AB), the Radius ſought.

The ſame may be done by the Line of Chords; for dividing 360 by 7, you find the Quotient to be 51° 26′ neareſt. Then you have a Chord BC, and its Degrees 51° 26′: Therefore by the 2d of this Chapter you may find the Radius.

The Uſes of theſe are frequent among Inginiers in fortifying, and drawing the Plans of fortified Towns. They know the Diſtance of the Angles of the Baſtions ought to be within Musket-ſhot; that is, they know the Diſtance of the Baſtions or Side of the exterior Polygon, which ſuppoſe to be GH: to compleat the Polygon, firſt find the Radius of the Circle (by the laſt) which will circumſcribe ſuch a Polygon. Then from G and H with that Radius deſcribe Arches cutting one another; this will give the Center of the Circle; which deſcribe, and then compleat the Polygon.

CHAP.

CHAP. XI.

Some Uses of the Line of Sines.

I. ANY Radius A B being given; to find the Lines answering to (35) any Number of Degrees.

Make (AB) the given Radius a Parallel at 90, and the Parallel at (35) the given Degrees, is (CD), the Line sought.

II. A Sine (CD) and its Degrees (35) given; to find the Radius.

Make the Sine (CD) a Parallel at (35) its Degrees; and the Parallel at 90, is (AB), the Radius sought.

III. A Sine (CD) and the Radius (AB) being given; to find the Degrees to the Sine.

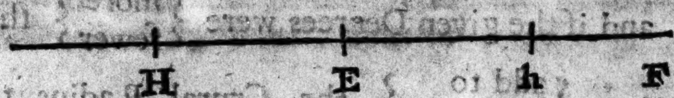
Make

Make (AB) the Radius a Parallel at 90; then move (CD) the Sine, along the Lines, till the Points of the Compasses fall on the same on each Side; or, which is the same Thing, move (CD) the Sine till it becomes a Parallel; which you will find to be at (35) the Degrees answering to the Sine.

IV. If you are to deal with a Sine so near the Center, that it is not of more than 5 Degrees, instead of it you may take the Chord as taught in the last Chapter. For under 6° 15′ the Chords and Sines so nearly agree, that their Differences are less than the 6000th Part of the Radius; and in a Sector whose Leg is a Foot, the Differences are less than the 500th Part of an Inch.

N.B. *In the first and third Examples, if the given Radius had been equal to that on the Leg, the Sine sought in the first would have been had by taking the Degrees from the Center; and the Degrees sought in the 3d, by laying the Sine from the Center upwards. And this without opening the Sector.*

V. To find the *versed Sine* of any Number of Degrees, whether of (55) less than 90, or of (125) more than 90; the Radius (EF) being given.



Find (EH or Eh) the right Sine of (35), the Difference between (55 or 125), the Degrees given, and 90 Degrees. And when the Degrees given are $\left\{\begin{array}{l}\text{fewer}\\ \text{more}\end{array}\right\}$ than 90 Degrees, $\left\{\begin{array}{l}\text{subtract}\\ \text{add}\end{array}\right\}$ $\left\{\begin{array}{l}\text{Eh}\\ \text{EH}\end{array}\right\}$ the right Sine found $\left\{\begin{array}{l}\text{from}\\ \text{to}\end{array}\right\}$ (EF) the Radius given; and you will have the *versed Sine* sought $\left\{\begin{array}{l}\text{hF}\\ \text{HF}\end{array}\right\}$.

VI. If a versed Sine, and the Radius were given, to find the Degrees answering to it.

Make the Difference between the Radius and the versed Sine, a right Sine to the same Radius; and find its corresponding Degrees. Then if the versed Sine were $\left\{\begin{array}{l}\text{greater}\\ \text{less}\end{array}\right\}$ than the Radius; $\left\{\begin{array}{l}\text{add to}\\ \text{subtract from}\end{array}\right\}$ 90 Degrees; and those Degrees being found, you have the Degrees sought.

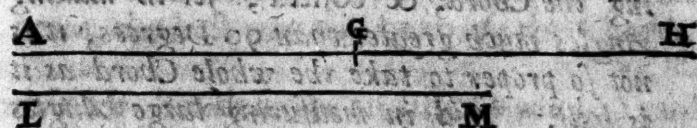
VII. If the Degrees answering to a versed Sine, and that versed Sine are given; to find the Radius.

Take the right Sine of the Difference of the given Degrees, and 90 from the Leg; and if the given Degrees were $\left\{\begin{array}{l}\text{more}\\ \text{fewer}\end{array}\right\}$ than 90, $\left\{\begin{array}{l}\text{add to}\\ \text{take from}\end{array}\right\}$ the Crural Radius this

right

right Sine; and opening the Sector make this $\left\{\begin{matrix}\text{Sum}\\ \text{Difference}\end{matrix}\right\}$ a Parallel at 90 on the Sines. Lastly take the given versed Sines in the Compasses, and carry them along the Lines till they rest on like Sines; then from the Sine where they rest to the Center is the Radius sought.

VIII. The Radius (GH) or (its double AH) the Diameter being given; to find the Chord of (94), or any Number of Degrees, even to 180.



Make the given Radius (GH) a Parallel at the Sine of 30; or the Diameter (AH) a Parallel at 90. Then the Parallel at (47), half (94) the given Number of Degrees, is (LM) the Chord sought.

IX. A Chord (LM), and (94) its Degrees being given; to find the Radius or Diameter.

Make on the Sines (LM) the Chord a Parallel at (47) half the Degrees given; then will the Parallel 30 be (GH) the Radius, and at 90 (AH) the Diameter.

X. The Radius (GH), or the Diameter (AH) being given, and also (LM) a Chord; to find the Degrees answering to the Chord.

Make (GH) the Radius a Parallel at 30, or (AH) the Diameter at 90; then carry (LM) the Chord along the Lines till it become a Parallel; and the double of (47) the Degrees where it is so, gives (94) the Degrees sought.

Tho' the three last Articles are Geometrically true, even to 180 *Degrees in finding the Chord*, & contra; *yet in making Angles much greater than* 90 *Degrees, it is not so proper to take the whole Chord as it is half. And in measuring large Angles use the Method laid down in the Use of the* Line of Chords.

XI. To open the Sector, so that the *Lines of Lines, Sines, Chords, Tangents*, each may make a right Angle with his like.

So open the Sector, that on the Lines 10 may reach from 6 to 8;

Or that on the Sines 90, may reach from 45 to 45;

Or from 40 to 50;

Or from 30 to 60;

Or from any Degrees to their Complement;

Or that on the Sines 45, may reach from 30 to 30.

XII. To

XII. To open the Sector, so that the aforesaid Lines may make (86), any Angle requir'd with their likes.

Take (43) half the given Degrees from the Sines on the Leg, and make it a Parallel at 30 on the Sines; it is done.

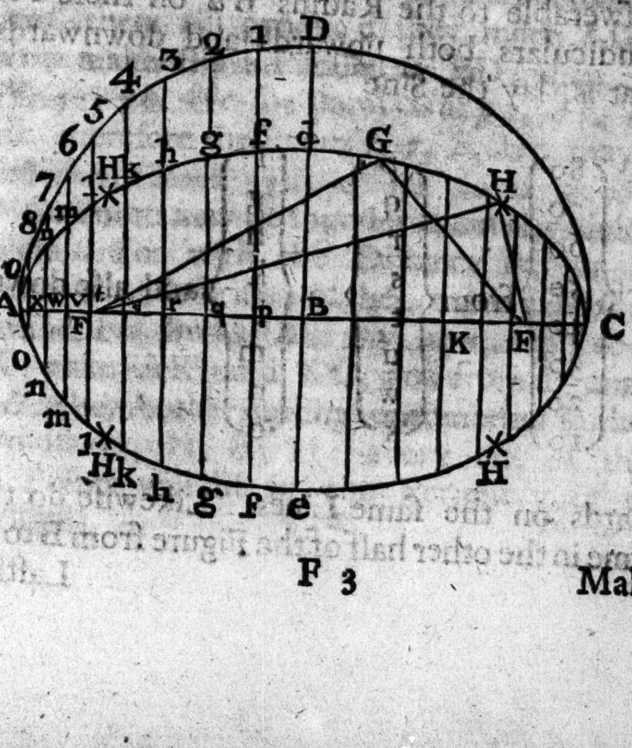
XIII. The Sector being open'd, to find the Angle that any of the aforemention'd Lines makes with his like.

The Parallel at 30 measur'd on the Lines, gives half the Degrees of the Angle.

In the two last Examples; if the Angle to be made or measured doth not exceed 60 Degrees, the best way will be to use the Chords.

XIV. To describe an Ellipsis whose Transverse and Conjugate Axes are given.

Let these Axes AC, *de*, cut each other into two equal Parts at right Angles in B.



 Make

Make AB a parallel Radius on the Sines; that is, open the Sector till AB reach from 90 to 90. And from that Point B lay towards A the Sine

of $\left\{\begin{array}{c}10\\20\\30\\40\\50\\60\\70\\80\end{array}\right\}$ to $\left\{\begin{array}{c}p\\q\\r\\s\\t\\u\\w\\x\end{array}\right\}$; and at the ſame time you may do the like in the other half *d* G *e*. Thro' every one of theſe Points draw Perpendiculars to AC, both upwards and downwards. Then make B*d*, half the conjugate Axis, a parallel Radius on the Sines; and lay the complement Sines of the preceding Degrees, anſwerable to the Radius B *d* on theſe Perpendiculars both upwards and downwards; that is, lay the Sine

of $\left\{\begin{array}{c}80\\70\\60\\50\\40\\30\\20\\10\end{array}\right\}$ from $\left\{\begin{array}{c}p\\q\\r\\s\\t\\u\\w\\x\end{array}\right\}$ to $\left\{\begin{array}{c}f\\g\\h\\k\\l\\m\\n\\o\end{array}\right\}$ and alſo downwards on the ſame Lines. Likewiſe do the ſame in the other half of the Figure from B to C.

Laſtly,

Lastly, Do the like for as many of the intermediate Degrees as you shall think sufficient; and thro' these Points *f*, *g*, *h*, *k*, *&c.* draw a crooked Line smoothly, and you will have the Ellipsis required.

If the Distances *p*, *q*, *r*, *s*, *&c.* had been the Sines of 15, 30, 45, 60, *&c.* answering to the Measures of Time in Hours; and if the Intermediates had been $7\frac{1}{2}$, $22\frac{1}{2}$, $37\frac{1}{2}$, *&c.* answering to the Time in half Hours; and others between these to Time in Quarters: you would have had that Ellipsis in that curious Problem, the Construction of a Solar Eclipse: In which case there would have been no need of drawing the Periphery, the Points thro' which it is to pass are sufficient.

Points thro' which the Ellipsis is to pass may be found by the *Line of Lines*. For on B, with the Distance AB, describe the Semicircumference ADC. Take *p*, *q*, *r*, *s*, *t*, *&c.* at pleasure, and draw Perpendiculars thro' them till produc'd they meet ADC: Now make B*d* a Parallel at BD; then will *pf*, *qg*, *rh*, *&c.* be Parallels at *p* 1, *q* 2, *r* 3, *&c.* therefore the Points *f*, *g*, *h*, *&c.* may be found.

Or Points may be found without any divided Scale. For with the Distance BC or AB from *d*; describe Arches cutting AC in F and F. Break BF any how, suppose in K; take AK and KC, and from F and F describe Arches cutting one another in the 4 Points H, H, H, H; every one of these 4 Points will be in the Periphery of the Ellipsis.

In like manner break BF in another Place, and you may find 4 other Points: And so as many as you please.

But of these three Methods the first is the best. It is better than either of the others on account of the great Affinity there is between the Circle and Ellipsis; there being no Property of the Ellipsis but what is also one in the Circle.

Each of the Points F is call'd a Focus; and they have this Property, *viz.* If to any Point whatsoever, suppose G or H, two Lines be drawn to those Points F, F; those Lines GF, GF together, or the Lines FH, FH together; are every where equal to AC.

Therefore Gardeners when they would strike an Ellipsis on the Ground, after they have mark'd out the Transverse and Conjugate Axes AC, *d e*, cutting one another at right Angles in B; and with the Distance BC from *d* have describ'd Arches cutting AC in F and F; they stick Pegs in the Points F, F, *d*, and tie a String gently strain'd round them; this String will be represented by the Lines FF, FG, GF.

Then taking up the Peg at *d*, move towards H marking on the Ground the crooked Line *d* H, the String being stretch'd with the same Tension as it was at *d*. At length the Peg will come to H; and by continuing the Motion, will describe HC, and come to C. And by proceeding till you come to *e*, A, and at last to *d*; and so the Ellipsis will be describ'd.

The

The Builders use a Tool for this Purpose call'd a *Trammel*, made by the Mathematical Instrument Makers.

If a Billiard Table be Elliptical, and if the Balls and the Table's Rim were perfectly elastick; if a Ball lay in one Focus, and there was a Net in the other, push the Ball which way soever you please, hard enough, it will come into the Net; and if this should strike 1, 2, 3, or more Balls, hard enough, no matter whether directly or obliquely, they would all come into the Net.

If Heat, Light, or any other Vertue, which will be so reflected that the Angle of Incidence be equal to that of Reflection; and this Virtue proceeding every way from one Focus, is reflected by the Periphery of the Ellipsis; they will all be collected in the other Focus.

Some Folio Volumes are insufficient to shew all the Properties of Conick Sections; nevertheless I thought it might yield a little Pleasure to the Reader to know some of them, in hopes that he might farther please and profit himself; most Mechanicks having Occasion sometimes, at least, to contemplate them, in order to form just Notions of what they are at work on.

XV. To describe a Parabola, whose Vertex and Focus is given.

Let the Vertex be K (Fig. 21. N° 1.) and the Focus D. Draw KD downwards, and it is the Axis. Make DA equal to DK. Make AK a parallel Radius at the Sine of 90. And divide

divide AK as a Line of Sines to every 10 Degrees, in B, C, D, E, F, G, H, I, to the Radius AK; and draw the Perpendiculars A *a*, B *b*, C *c*, &c. on both Sides of the Axis. To the Radius AK make I *i*, H *h*, G g, F *f*, &c. the Chords of 10, 20, 30, 40, &c. Degrees. And all the Points *i*, *h*, *g*, *f*, &c. will be in the parabolick Curve.

And if other intermediate Degrees are laid down, more Points may be found between these; and so the Curve more nearly exhibited to the Eye.

This is the Curve in which (abating the Resistance of the Air) all Projectiles move, as Cannon Balls, Bombs, Stones, &c. when flung forward.

A Bomb flung from *d* so as to go to *e*, would pass thro' the Points *f*, *g*, *h*, *i*, K, *i*, *h*, *g*, *f*, *e*, *d*, if the Ground was Horizontal. But if the Ground (N°. 2.) descended, the Ball would continue to go on in the Curve to *c*, *b*, *a*, &c. till it struck the Ground. And if the Ground ascended, it must meet the Ground before it came to *d*, perhaps at *e*, *f*, *g*, &c. (See N°. 3.)

But in every of the three Cases the Ball will move in a Parabola.

If a Part of half the Parabola GK *g*, should be turn'd round upon GK at rest, it would form a Concave like a Bowl-dish. If such a Concave be made of Metal, and very well polish'd, and G *g* be about 18 Inches; and if the Sun shining on this polish'd Concave be so turn'd about, that the Axis KG points to the Sun; all the Rays of Heat coming from

the

the Sun, and falling in this Concave, will be reflected to the Focus D: And there the Heat will be so intense, as to vitrify hard Stones, and to melt Steel, in less than a Minute. But the Concave itself will be no hotter than other the like Metal expos'd to the Sun.

The Parabola might have been describ'd by the *Line of Lines.* For drawing PK perpendicular to the Axis KA. And taking the Points L, M, N, O, &c. at pleasure, and raising the Perpendiculars L*i*, M*b*, N*g*, &c. Find a third continual Proportional (as in the 15th Example of the Use of the *Line of Lines*) to the double of AK,

$$\text{and}\left\{\begin{array}{l}KL\\ KM\\ KN\\ KO, \&c.\end{array}\right\}\text{and you will get}\left\{\begin{array}{l}L\,i\\ M\,b\\ N\,g\\ O\,F, \&c.\end{array}\right\}$$

$$\text{which Distances laid from}\left\{\begin{array}{l}L\\ M\\ N\\ O, \&c.\end{array}\right\}\text{to}\left\{\begin{array}{l}i\\ b\\ g\\ f, \&c.\end{array}\right\}$$

will give those Points in the Curve.

The Parabola also may be laid down without any divided Lines. But the first Method here proposed I recommend as the best.

If

If thro' the three Points *i* K *i* be describ'd an Arch of a Circle, whose Center is very near to A; that circular Arch will have very nearly the same Properties with the small parabolical Arch *i* K *i*: And because it is easier to polish a Glass or Metal, that is in a spherical Form than any other, the Philosophers often use the Spherical one instead of that formed by the Rotation of the Parabola on its Axis.

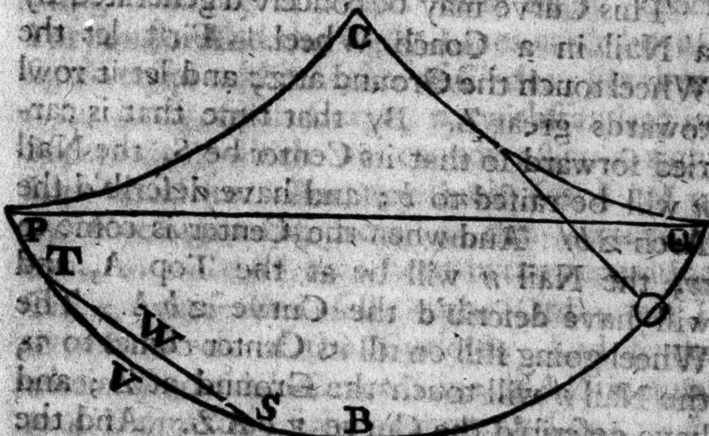
XVI. To describe a Cycloid whose Height, that is, the Diameter, of its generating Circle is given.

Let its Height be AB 77 (Fig. 22); and let the Line on which the Cycloid is to be describ'd, be *z* BZ; then will the Semicircumference of the Circle AHB be 121; which lay from B to Z, and from B to *z*. Divide the Semicircumference AHB into any Number of equal Parts, the more the better, suppose 12, in the Points B C D E F G H I K L M N; and divide the half-Base BZ into the like Number of equal Parts. Thro' B, C, D, E, &c. draw Lines parallel to BZ on both Sides of AB, and lay BO, BP, BQ, BR, &c. from N to *n*, from M to *m*, from L to *l*, from K to *k*, &c. And do the same on the other Side of the generating Circle; so will the Points *c*, *d*, *e*, *f*, *g*, *h*, &c. be all in the Curve of the Cycloid.

If this Curve be inverted, as in Numb. 2. and a Body descends from P, T or S, by its own Gravity, to B the lowest Point

Point and Vertex of the Curve; the Times of these Descents shall all be equal.

Numb. 2.



If a Body descends from T to S in the Curve TVS by its own Gravity; the Time of its Descent will be less than if it went in the strait Line TWS; or less than if it went in any other Line: therefore this is call'd the *Line of swiftest Descent*.

If the Curve QBP were cut into two equal Parts in the Vertex B; and those Parts QB, PB, were turn'd upwards above the Line QP, so that P and Q of P and Q become C; and also the convex Parts of these Curve Lines towards one another; and if Plates were bent in the Form of those crooked Lines, and a Pendulum hung at C, of a length equal to the crooked Line BQ, counted from the

Point

†

Point of Suspension to the Center of Oscillation: I say that Center of Oscillation will describe the Cycloid PBQ; and all the Vibrations, whether long or short, shall be perform'd in the same time; therefore it is call'd the *Isochronal Curve*.

This Curve may be conceiv'd generated by a Nail in a Coach Wheel. First let the Wheel touch the Ground at *z*, and let it rowl towards great Z. By that time that is carried forward so that its Center be S, the Nail *n* will be raised to *b*; and have describ'd the Arch *zb*. And when the Center is come to *w*, the Nail *n* will be at the Top A, and will have describ'd the Curve *zb* A. The Wheel going still on till its Center comes to *v*, the Nail *n* will touch the Ground at Z; and have describ'd the Curve *zb* AZ. And the Circumference of the Wheel or generating Circle would have measur'd out Z*z* the Base of the Curve. And so the Base of the Curve is equal to the Circumference of the generating Circle.

If the Wheel should continue thus to rowl on *ad infinitum*, an Infinity of such crooked Arches would be describ'd, which all consider'd together, is but the same Curve continu'd; and so this Curve by Mathematicians is said to be a *Line of the infinite Order*; for it may be cut by one right Line into an Infinity of Points.

No Algebraick Equation can express the Relation of its Ordinate to its Abscis; therefore it is call'd a *Transcendent Curve*. And notwithstanding this, its Area is comparable with

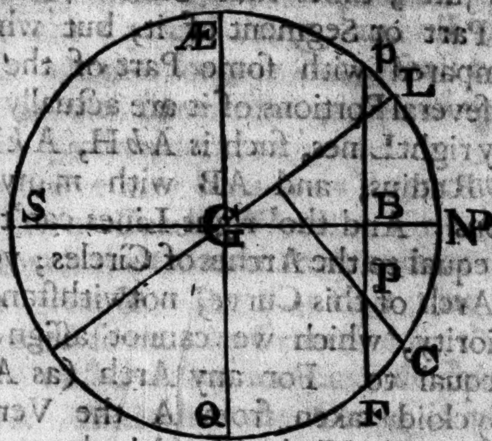
with the Area of its generating Circle, and is indeed just 3 times that Circle. And there is no Part or Segment of it, but what may be compared with some Part of the Circle. Nay, several Portions of it are actually squareable by right Lines, such is A *b* H, A *k* B; A *w* being Radius, and AB with many other Portions. And tho' right Lines can't be assign'd equal to the Arches of Circles; yet there is no Arch of this Curve, notwithstanding its Superiority, which we cannot assign a right Line equal to. For any Arch (as A *l e*) of the Cycloid taken from A the Vertex, is equal to twice AL the Chord in the generating Circle, whose End falls in the Ordinate *e* E to that Arch of the Cycloid. And so the whole Cycloidal Curve Line *z* A Z is equal to 4 Diameters of the generating Circle.

XVII. To solve by the Lines of Sectoral Sines the grand Geographical Problem; that is, having the Latitudes of two Places, and their Differences of Longitude, to find their Distance.

Let the Example be of *London* in the Latitude of 51$^{\text{d}}$ 32$'$ N, and of *Pekin* in *China*, in the Latitude of 40° N; their Difference of Longitude is 117 ∟ 30.

With any Radius (See the following Fig.) describe the Circle NP Æ SQ, draw NP G, and lay the Chord of 90 from NP to Æ, and draw Æ G Q. Lay the Chord of 51 ∟ 32 (*London*'s Latitude) from Æ to L and draw LG; lay the Chord of *Pekin*'s Latitude (40 ∟ 0) from Æ to P and from Q to F.

2 And

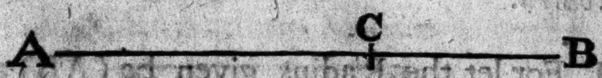


And draw p F; make B p on the Sines a parallel Radius. Then take the Sine of the Difference of 90 Degrees and Difference of Longitude (27° ∟ 30′) and lay it from B to P; if the Difference of Longitude be $\left\{\begin{matrix}\text{less}\\ \text{greater}\end{matrix}\right\}$ than 90 Degrees from B towards $\left\{\begin{matrix}\text{P}\\ \text{F}\end{matrix}\right\}$ call it P. That is the Place of *Pekin*. From P draw PC perpendicular to LG; then is LC the Distance sought, which measure by the Chords.

The two Astronomical Problems are performed after the same Manner; *viz.* when the Latitudes of two Stars, with their Differences of Longitude; or the Declinations of two Stars with their Differences of Ascension are given; to find their Distance.

XVIII. To

XVIII. To cut a given Line AB into mean and extream Proportion. Make AB a Parallel at the Chord of 60. Then is the Parallel at the Chord of 36 the greater Segment AC. Or make AB a Parallel at the Sine of 54; then is the Parallel at 30 the greater Segment AC; and the Parallel at 18 the lesser Segment BC. *Et vice versa.*



CHAP. XII.

Some Uses of the Tangents *and* Secants.

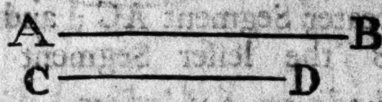
THERE are two Lines of Tangents on the Sector, one (like the Chords) beginning at the Center, and running to 45 Degrees, where it is equal to the Radius, as is demonstrated in the 3d Chapter: The other which I call the upper Tangents, beginning with 45 Degrees (which therefore is the Radius) at the Distance of the Number 25 of the Lines

 from

from the Center, and running to upwards of 70, *viz.* to as many Degrees as will come on.

By the former may be found the Tangent of any Number of Degrees not more than 45, to any Radius within the Compaſs of the Sector, & *contra.*

I. For let the Radius given be (AB) any Line; to find the Tangent of (37° 15′) any Number of Degrees.



Make (AB) the Radius given a Parallel at 45; and then the Parallel at (37° 15′) the Degrees given, is the Tangent (CD) ſought.

II. A Tangent (CD) being given of (37° 15′) a given Number of Degrees; to find the Radius.

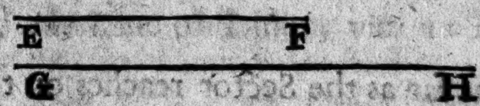
Make (CD) the Tangent given a Parallel at (37∟15) the Degrees given; and then will the Parallel at 45 be the Radius ſought.

III. A Tangent (CD) and the Radius (AB) being given; to find the Degrees anſwering to that Tangent.

Make the given Radius (AB) a Parallel at 45°; then the Tangent (CD) carry along the Lines till it becomes a Parallel; that is, till

till the Points of the Compasses rest on the same Tangents on such Line; which you will find to be at 37° 15.

IV. The Radius (EF) being given; to find the Tangent of (57°) any Number of



Degrees above 45, as far as the Sector reaches on the upper Tangents.

Make EF a Parallel at 45° on the upper Tangents (commonly call'd the *lesser Tangent*, on account of the lesser crural Radius) and the Parallel at 57 is (GH) the Tangent sought.

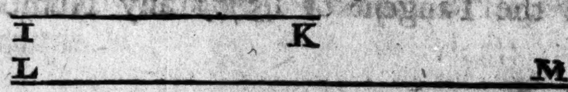
V. A Tangent GH and its corresponding Degrees (57) above 45 being given; to find the Radius.

Make CD the given Tangent a Parallel at (57) the Degrees given, and then will the Parallel at 45 be the Radius sought.

VI. A Tangent (GH) and the Radius (EF) being given; to find the Degrees answering to this Tangent.

Make the Radius (EF) a Parallel at 45; and then the Tangent GH carried along the Lines till it becomes a Parallel, will give (57) the Degrees required.

VII. The Radius (IK) being given; to find the Secant of (57.°) any Number of De-



grees as far as the Sector reaches on the Line of Secants.

Make (IK) the Radius a Parallel at the Beginning of the Divisions; that is, at the Secant of o Degrees; and the Parallel at (57) the Degrees given, will give (LM) the Secant required.

VIII. A Secant (LM) and (57) its corresponding Degrees being given; to find the Radius.

Make (LM) the Secant a Parallel at (57) the Degrees given; and the Parallel at o Degrees will be the Radius sought.

IX. The Radius (IK) and the Secant (LM) being given; to find the Degrees answering to that Secant.

Make (IK) the Radius a Parallel at o Degrees; and carry (LM) the Secant along the Lines until it becomes a Parallel; which will be at (57) the Degrees sought.

It is to be observ'd, that as the crural Radius of this Line of upper Tangents, and also of the Secants, were made but to ¼ Part of the Line

Line of Lines, *in order to bring on as many of the upper Degrees as we could conveniently; the Radius is here but 2 Inches and $\frac{119}{128}$; and so its double is 5 Inches and $\frac{55}{64}$, which is the greatest Radius that these Secants and upper Tangents can simply be applied to; and so the greatest Circle that they can simply be applied to, is that whose Diameter is equal to the Length of the* Line of Lines; viz. 11 *Inches and $\frac{23}{32}$.*

Had it been made much larger, there had been wholly lost the Use of the upper Degrees; had it been made much less, the Detriment would have been greater than the Gain.

But tho' the Tangents and Secants reach but to little more than 75 Degrees, suppose to 76, yet by the same Sector the Tangent and Secant of any Number of Degrees may be had.

X. To find the Tangent of (82°) any Number of Degrees above 76, not exceeding (83) the middle Degree between 76 and 90.

From 90 subtract (82) the Degrees, and from (82) the Degrees given, subtract (8) the preceding Remainder; and there will be left 74: Then the Sum of the Tangent and Secant of (74), this last Result will be the Tangent of (82) sought.

XI. A Tangent of (82) any Number of Degrees being given; to find the Secant of the same.

To the Tangent of (82) a given Number of Degrees, add the Tangent of half (8) the Complement of (82) those given Degrees to

 90,

90, (*viz.* the Tangent of 4); and you will have the Secant of (82) the given Number of Degrees.

And now the Sum of the Secant and Tangent of 82, is (as in N. 10.) the Tangent of 86°. And if to this you add the Tangent of half 4, *i. e.* 2; you have the Secant of 86 by N. 11. In like manner you may proceed to find the Tangent and Secant of 88, 89, 89 $\frac{1}{2}$ Degrees.

XII. To find the Tangent and Secant of any Number of Degrees beyond the Length of the Sector; suppose of 87.

Take 87 from 90, and there remains 3; take 3 from 87, and there is left 84. Now, if you had gotten the Tangent and Secant of 84, the Tangent and Secant of 87 would have been gotten (by N. 10. and 11.) But 84 is not on the Sector.

Take 84 from 90, and there remains 6; take 6 from 84, and there is left 78. Now, if you had gotten the Tangent and Secant of 78, the Tangent and Secant of 84 might be had by the 10th and 11th. But if 78 be not on the Sector;

Take 78 from 90, and there remains 12; take 12 from 78, and there is left 66; whose Tangent and Secant are on the Sector.

Therefore (by N. 10. and 11.) from the Tangent and Secant of 66, you may find the Tangent and Secant of 78.

And from the Tangent and Secant of 78, you may find the Tangent and Secant of 84.

And

And then from the Tangent and Secant of 84 you may find the Tangent and Secant of 87 sought. The like may be done of any other Tangent and Secant.

XIII. The Sector cannot be so open'd as to make the Radius on the Secants or upper Tangents quite 6 Inches. But this Defect may be very easily cured by the Sines and the lower Tangents. For let it be requir'd to find the Tangent and Secant of 65 Degrees, to a Radius of 6 Inches. Make 6 Inches a Parallel at the Tangent of (25) the Complement of (65) the Degrees given: Then the Parallel at 45 gives the Tangent sought.

Make 6 Inches a Parallel at the Sine of (25) the Complement of (65) the Degrees given. Then the Parallel at 90 is the Secant sought. In like manner the Tangent and Secant of Degrees beyond the Rule may be had if a lesser Radius would suffice. For to a Radius of 3 Inches may be had the Tangent and Secant to 82 $\frac{1}{2}$ Degrees, at one opening of the Sector. And if 2 Inches will suffice for the Radius, you may have the Tangent and Secant to 85 Degrees at one opening of the Sector. For make 2 Inches a Parallel on the Sines at 5 Degrees the Complement of 85; and then the Parallel at 90 is the Secant sought. And make 2 Inches a Parallel on the Tangents at 5 Degrees, then the Parallel at 45 is the Tangent of 85 sought. And the Converse of these, *viz.* the finding of the Degrees answering to a Tangent or Secant when the Radius given is greater than the Radius

on the Secants or upper Radius is equally eaſy. For make the given Tangent a Parallel at 45 on the lower Tangents; and where the given Radius is a Parallel on the Tangents, you have the Complement of the Degrees ſought. Or make the given Secant a Parallel on the Sines at 90; and where the given Radius is a Parallel on the Sines you have the Complement of the Degrees ſought.

Therefore the Degrees anſwering to any given Tangent or Secant not longer than 23 Inches $\frac{7}{16}$ to a Radius given leſs than them; may be found by the forementION'd Sector. Or,

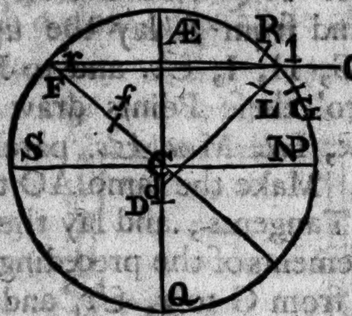
Laſtly, If the Secant or Tangent, and their Degrees were given, the reſpective Radius would be found. For the Tangent given being made a Parallel at 45 on the Tangents; the Parallel at the Tangent of the Complement of the Degrees is the Radius ſought. And the Secant being made a Parallel at the Sine of 90; the Parallel at the Sine of the Complement of the Degrees is the Radius ſought.

XIV. The Latitudes of two Places, together with their Difference of Longitude being given; to find their Diſtance at one opening of the Sector.

Let the Example be for *London* and *Rome*, whoſe Latitudes are 51° 30′ N, and 41° 50′ N; and Difference of Longitude 12° 40′.

With

With any Radius describe a Circle ♈ ÆQ, and quarter it by drawing ♈ S and ÆQ at



right Angles to one another. By help of the Chords (the Opening of the Sector remaining the same) lay off the Latitude of *London* and *Rome* 51 30 and 41 50 from Æ to *l* and R; and also of *Rome* from S to *r*, and draw *l r*. Lay the Tangent of the Difference of Longitude 12° 40′ from C to D, and the Tangent of its half from C to *d*. And draw *d l*. with the Distance D ♈ from D, describe an Arch cutting *d l* in L. Bisect the Distance R L at right Angles with the strait Line *c* F. Produce *c* F till it meet C *r* in F. Measure C F on the Tangents, and lay the Tangent of half those Degrees from C to *f*. Lastly, a Ruler laid from *f* to L meets ♈ Æ in G. And the Distance GR is the Distance sought; which measure on the Chords.

XV. To describe an Hyperbola, the Assymptotes and Vertex being given.

Let

Let A B, A C, (Fig. 29.) be the Assymptotes, and E the Vertex of the Curve. From E draw E O, E Q parallel to the Assymptotes. Make A O or A Q a Radius on the upper Tangents; and from A lay the upper Tangents to F, G, H, I, &c. and to K, L, M, &c. and thro' these Points draw F*f*, G*g*, H*h*, &c. K*k*, L*l*, M*m*, &c. parallel to the Assymptotes. Make the same AO the Radius on the lower Tangents; and lay the Tangents of the Complements of the preceding Tangents from F to *f*, from G to *g*, &c. and also from K to *k*, from L to *l*, &c. Then will the Points E, *f*, *g*, *h*, &c. *k*, *l*, *m*, &c. be Points in the Hyperbolick Curve. Consequently a crooked Line smoothly drawn thro' those Points, (especially if many of them are taken, suppose to every Degree) will exhibit the Curve very beautifully to the Eye.

But Points may be found thro' which the Hyperbola may pass without any divided Scale.

This Curve the Hyperbola continually approaches its Assymptotes, but never meets them.

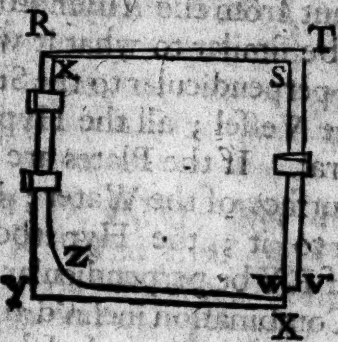
In any Part of the burning Zone, or in any Part of the temperate Zones, if a Staff be any how stuck in a plain level Ground, whether upright or leaning, the End of the Shadow of the Staff shall describe an Hyperbola every Day in the Year.

In all Dials whose Stiles height is less than 66°L30′, the Parallels of Declination are Hyperbola's. And most part of the beautiful Ornaments drawn on Dials depend on them.

By

By Nature also in this Curve doth Fluids rise to any heighth above the common open Level.

Let RYXS, RYVT be two square Plates of Glass pinch'd close on one Side of RY by two



bits of bent Tin lapping round the Glasses; and open'd on the opposite Side to about a quarter of an Inch by a small wooden Wedge; the Wood gently giving way a little, the Wedge soft not breaking the Glass. If this Combination be plac'd in a flat Bottom, with Water about $\frac{1}{10}$ of an Inch deep, or more or less at Pleasure; you will see the Water rise up to the Top RST, at the close Side RY, in the Form of the Figure RXZWXYR; and the Curve XZW will be an Hyperbola; RYX its Assymptotes; and Z its Vertex.

If the Liquor be tinged, and the Inside of the Glass Plates made very plain, and wetted before you try the Experiment, the Phænomenon will be more pleasant.

This

This Truth is proved by those who shew the Philosophical Experiments; from the rising of Fluids in capillary Tubes.

If the wooden Wedge be drawn out a little so that the Plates come nearer together, the Vertex and every Part of the Curve will be farther distant from the Assymptotes. But the Curve will be similar to what it was before.

If RY be perpendicular to the Surface of the Water in the Vessel; all the Hyperbola's will be Equilateral. If the Plates are perpendicular to the Surface of the Water; but the Line RY inclin'd to it; the Hyperbola's will be Scalene. If RY be perpendicular to YZ; and the whole Combination inclin'd to the Surface of the Water; the Hyperbola's will be farther remov'd from the Assymptotes.

XVI. To describe the true Sea-Chart, sometimes call'd *Wright's* or *Mercator's*. And let the Example be from the Latitude of 45 to the Latitude of 55 Degrees.

Open the Sector so that the Distance of the Beginning of the Secants may represent one Degree of Longitude. And having drawn AM (Fig. 31.) to represent the Parallel of 45 Degrees; lay on it the aforesaid Distance (*viz.* the Parallel at 45) from A to B, from B to C, from C to D, &c. as far as you please. And draw A X at right Angles to A B. The Sector open'd as before; take the Parallel Secant of 45 ½, and lay it from A to N; and the Parallel at 46 ½, and lay it from N

to

to O. And in like manner the Parallel

at	$47\frac{1}{2}$	and lay it from	O to P
	$48\frac{1}{2}$		P to Q
	$49\frac{1}{2}$		Q to R
	$50\frac{1}{2}$		R to S
	$51\frac{1}{2}$		S to T
	$52\frac{1}{2}$		T to V
	$53\frac{1}{2}$		V to W
	$54\frac{1}{2}$		W to X

Now thro' N, O, P, Q, *&c.* draw Lines parallel to A B. And thro' B, C, D, *&c.* draw others parallel to A X. Then in ſome convenient Place of the Draught lay down the Rumbs, as taught in the 18th Article of the 10th Chapter; and you will have the true Chart requir'd, divided to every whole Degree.

If greater Exactneſs be required, make the parallel Radius on the Secants equal to half a Degree of Longitude; that is, take the Diſtance from 45 to 45, and lay from A to B, from B to C, from C to D, *&c.*

Then take the Diſtances at the Middle of every half Degree, *viz.* at $45\frac{1}{4}$, $45\frac{3}{4}$, $46\frac{1}{4}$, *&c.* and lay them from A to N, from N to O, *&c.*

And thro' theſe Points draw Lines parallel to A B and A X; and you will have the Parallels of Latitude for every half Degree's Diſtance; and the Meridians at every half Degree of Longitude diſtant from one another.

The Parts of a Degree leſs than a half, ſuppoſe Quarters, are had in Longitude by Biſection. The ſame may be done for the farther Diviſion on the Meridian, near enough for any Uſes.

If the Foot Sector beforementioned be here made uſe of, a Degree of Longitude may be made 11 Inches; and ſo every League will be repreſented by more than half an Inch. Which is more than ſufficient for any Degrees of Exactneſs required in keeping an Account of a Ship's Way, and pricking the Ship's Way down on that Chart. For indeed there is no Neceſſity for laying down every half Degree; unleſs it be for Voyages very near the Pole. But divide every whole Degree of the Meridian into 20 equal Parts, to repreſent Leagues. And the ſame in ſome one Parallel of Latitude; or on the Equinoctial.

This Chart differs from the plain Chart and the Globe in theſe following Particulars.

The Meridian in this Chart is a ſtrait Line divided by unequal Diſtances repreſenting Degrees; and theſe unequal Diſtances continually increaſe as they come nearer the Poles. In the plain Chart the Meridian is a ſtrait Line equally divided into Degrees. On the Globe the Meridian is a great Circle equally divided into Degrees.

The Parallels of Latitude in this Chart, are all ſtrait Lines of the ſame Bigneſs, and are all alike equally divided; but theſe Parallels are at unequal Diſtances from one another. But a Degree of one of theſe Parallels bears ſuch Proportion to a Degree of the Meridian

in the Latitude of this Parallel; as a Degree of the ſame Parallel of Latitude on the Globe, is to a Degree of the Meridian on the Globe. But tho' the Parallels of Latitude in the plain Chart are all ſtrait Lines of the ſame Bigneſs, and alike divided; they do not bear that Analogy to their correſpondent Degrees of Eaſting and Weſting, as thoſe on the Globe do.

This Chart and the plain Chart agree only in this, *viz.* the Rumbs in both are ſtrait Lines; and the like Rumbs make equal Angles with every Meridian they meet. But on the Globe the Rumbs are crooked Lines of the ſpiral Kind reſpecting the Pole as their common Eye; one of which they continually approach: But cannot be ſaid to come quite to it without an indefinite Number of Times running round it.

This Chart at its firſt Appearance was, and ſtill is look'd upon as the fitteſt of all others for thoſe who ſail long Voyages; eſpecially if they are oblig'd to be in high Latitudes.

The Problems uſually perform'd by this Chart are theſe following.

Of any two Places laid down on the Chart, to find their Latitudes, Difference of Longitude, the Rumb from the one to the other; and their true Diſtance in that Rumb.

And of theſe 4, any two being given, the other two may be found; ſave that when the Difference of Longitude and Diſtance are the two given; the Operation will be tentative.

And theſe 16 Problems are by this Chart better ſolv'd than they are by the Globe itſelf.

Of

Of these Solutions, together with the Comparison of this Chart with the plain Chart, I have not at present Room enough to give Examples.

CHAP. XIII.

The general Use of the Line of Numbers.

THE Reading, Estimating, and Valuing the several Divisions and Distances on this Line were fully laid down in the 7th Chapter; therefore need not be repeated here.

The Operations perform'd by this Line are these following; viz. *Multiplication, Division, Proportion, Extraction of Roots; and several others depending on these.*

I. The general Rule for working Proportion is, Extend the Compasses from the first Number to the second; and the same Extent laid the same Way, will reach from the third to the Answer.

N.B. By the same Way, I mean, that if the Compasses, open'd from the first to the second, was from the Left-Hand to the Right, that is upwards, then they should be applied from the Left to the Right for the Answer. And on the other hand; if the Extent

Extent of the Compasses from the First to the Second, be from the Right to the Left; then the Compasses from the Third to the Answer, must be also from the Right to the Left.

Example 1. To the three given Numbers 40, 55, 56; to find a fourth Proportional. Extend the Compasses from 40 to 55; and that Opening will reach from 56 to 77, the Answer.

Example 2. To the three given Numbers 150, 90, 81, to find a fourth Proportional. Extend the Compasses from 150 to 90; the same will reach from 81 to 54, the Answer.

In the first of these Examples the Extent was made to the Right-Hand, or upwards; in the last, to the Left, or downwards. But we are to remember what hath been said elsewhere; *viz.* that of four proportional Numbers, either the Second and Third may be taken for the other. Note also, in taking of the Extent of the First to the Second, one Value may be assign'd to the Divisions; and in laying it from the Third to the Answer, the same, or any other Value may be assign'd.

Example 3. To the given Numbers 4, 9, 632, to find a fourth Proportional. Extend the Compasses from 4 to 9 (calling the Left-Hand 1 an Unite) and the same Extent (calling the same 1 an 100) will reach from 632 to 1422.

Example 4. To the given Numbers 12, 5, 420, to find a fourth Proportional. Extend the

Compasses from 10 to 5 (calling the middle 1 ten) and the same Extent (calling the middle 1 an 100) will reach from 420 to 175.

If the first Extent be either too long for your Compasses, or falls beyond the Line, be it upwards or downwards; vary (but let it be Decimally) your given Numbers, as is taught in the 4 first Variations in Chap. 8.

Example 5. To the given Numbers 2, 98, 50; to find a fourth Proportional. Here I vary them by multiplying the first by 10; and then they are 20, 98, 50. And the Answer comes out 245, which by the 2d Variation is 2450.

Example 6. To the given Numbers 2, ⌊3, ⌊73 to find a fourth Proportional. Calling the Middle an Unit, the Extent from 2 to ⌊3, will reach from ⌊7 to 1⌊05, the Answer.

II. Two Numbers being given; to find a 3d, 4th, 5th, &c. continually Proportional.

Extend the Compasses from the first to the second; and that Extent will reach from that 2d to the 3d Proportional; the same Extent will reach from the 3d to the 4th; from the 4th to the 5th, &c.

Example 1. Let the Numbers be 4 and 8; then the Extent from 4 to 8, will reach from 8 to 16, from 16 to 32, and from 32 to 64; but from 64 on the first Line to 128, and from 128 to 256, &c.

Example

Example 2. Let the Numbers be 10 and 7. The Extent from 10 to 7 will reach from 7 to 4 9, from 7 9 to 4 49, &c.

III. Multiplication is perform'd from this Consideration. Unity is to one Factor, as the other is to the Product.

Example 1. Multiply 33 by 25. Extend the Compasses from 1 to 25, the same Extent will reach from 33 to 825 the Product.

In this Example, I chang'd the Proportion by Variation of Chap. 8. *viz.* I made it as 1 is to 2, 5, so is 330 the Answer.

But the Products may be had without any Consideration of those Variations thus: Find both the Factors in the first Line of Numbers, without regard to the Value of the Units. Extend the Compasses from 1 to the least of the Factors; and that Extent will reach from the other to the Answer, which will always consist of as many Integral Places, as there were Integral Places in both Factors, if the Answer falls above the Middle 1. But will be a Place less, if it falls short of it.

Example 2. Multiply 575 by 28. Seek 28 in the first Line of Numbers, and also 575 without any regard to the Value of the Unit. Then the Extent from 1 to 28 will reach from 575 to 16100; *viz.* to the Number of the Integral Places in both Factors, because the Product falls above the Middle 1.

Example 3. Multiply 42⌊5 by 36. Extend the Compasses from 1 to 425; and the same Extent will reach from 36 to 1530⌊0. That is, to as many Integral Places, as there were Integral Places in both Factors.

Example 4. But if the Factors had been 425 and 16, the Product falls below the Middle 1; and therefore consists but of 4 Integral Places; *viz.* one less than 5 the Number of Integral Places in Factors.

The same had been equally true if one or both Factors had been pure Fractions; provided that the Cyphers, between the significant Figures and the Place of Unity, be esteem'd as Negatives. So if 22 were to be multiply'd by ⌊08. Extend the Compasses from 1 to 8, the same Extent will reach from 22 to 17⌊6.

IV. Division is perform'd from this Consideration. The Divisor is to Unity, as the Dividend is to the Quotient. Therefore the Extent from the Divisor to Unity, will reach from the Dividend, downwards to the Quotient.

Example 1. Divide 825 by 25. Extend the Compasses from 25 to 1, the same Extent will reach from 825 to 33 the Quotient.

But the Quotients may be had thus. Find both the Divisor and the Dividend in the second Line of Numbers, without regard to the Value of the Unit. Then the Extent

from the Divisor to the Middle 1; will reach from the Dividend downwards to the Quotient; which, if it falls below the middle 1, will consist of as many Integers, as the Integral Places of the Dividend are in Number more than those in the Divisor. If it reaches not to the Middle, the Quotient will consist of one more than the Excess.

Example 2. Divide 1530 by 36. These being both sought in the second Line, extend the Compasses from 36 to the Middle 1, and the same Extent will reach from 1530 to 425. But because it falls below the Middle 1, the Integral Places are 2, equal to the Difference between the Number of the Integral Places in the Dividend, and those in the Divisor.

Example 3. Divide 16100 by 28. These being found in the second Line, the Extent from 28 to the Middle 1, will reach from 16100, down to 575; which because it falls below an Unit, consists of 3 Integral Places; 3 being the Difference between 5 and 2.

Example 4. Divide 935 by 25. These being found in the second Line as before; the Extent from 25 to the Middle 1, will reach from 935 to 374; which because it falls above the Middle 1, will consist of one Integral Place more than the Difference of the Places in the Numbers given; therefore the Answer consists of two Integral Places; consequently it is 37⌊4.

What hath been said in Multiplication concerning the pure Decimals may be applied here.

V. To find a mean Proportional between two Numbers given. When the Numbers consist both of odd Numbers of Places, or both of even; seek them both in the second Line of Numbers. But when one hath an odd Number of Places, and the other an even; seek one in the first Line of Numbers, the other in the second. Then the Middle between these two Numbers gives the mean Proportional sought. And the Number of the Integral Places of this Mean shall be half the Sum of the Number of Integral Places in both the given Numbers, if that Sum can be halv'd. But if it be odd, and so cannot be halved exactly, it shall be half of one $\left\{\begin{matrix}\text{more}\\ \text{less}\end{matrix}\right\}$ when the Answer falls $\left\{\begin{matrix}\text{above}\\ \text{below}\end{matrix}\right\}$ the middle Unit.

Example 1. Find a mean Proportional between 252 and 468.

Because they are both of odd Places I seek them on the 2d Line, and I find their Middle 336, which is the Mean sought, and doth consist of 3 Integral Places, the half of the given 6 Places.

Example 2. Find a mean Proportional between 8 and 128.

Because

Because both are of odd Places, I seek both in the second Line, and their Middle gives me 32, which are both Integers, because 2 is the half of 4 the Number of Integral Places given.

Example 3. Find a mean Proportional between 40—5 and 84—50.

Because both are of even Places, I seek them in the second Line, and the Middle between them I find to be 585. Because there was but 4 Integer Places in both Numbers, the Answer is 58—5.

Example 4. To find a mean Proportional between 8 and 98.

I seek the 8 on the first Line, and the 98 on the second, according to the Rule. And because the middle Point falls above the middle Unit on 28; I add 1 to 3 the Number of Integers in the given Numbers, and half the Result 4 (*viz.* 2) shews the Number of Integers in the Mean sought.

Example 5. Find a mean Proportional between 544—5 and 12—5.

The Numbers being sought on both Lines according to the preceding Directions, I find the middle Point falls below the Middle 1 on 825. Now because the middle Point falls below the Middle 1, I subtract 1 from the Sum of the Numbers of the Integral Places in the given Numbers, and conclude that 2, half the Remainder, is the Number of the Integral Places in the

Mean sought. Therefore the Mean sought is 821. 5.

If one or both are purely Decimals, seek them as tho' they were Integers, by counting the Cyphers between the significant Places, and the Place of Unity as negative Integers. And in valuing the Mean consider the Number of these Cyphers negatively.

VI. To extract the Square Root of any given Number. Find a mean Proportional between that Number and Unity. It is the Square Root sought.

So the Square Root of 4225 is 65, the mean Proportional between 4225 and 1.

VII. To extract the Cube Root. The Integral Places of the Number given, consists of a Multitude which is either an exact Multiple of 3, or exceeds a Multiple of 3 by 1, or exceeds a Multiple of 3 by 2. If it exceeds a Multiple of 3 by 1, seek it in the first Line; if by 2 in the second. Divide the Distance between the Number and the first 1 into 3 equal Parts; and that Point of Division which falls next the first 1 is the Cube Root sought.

So 3 is the Cube Root of 27, 2 of 8, and 4 of 64. When the Places are in Multitude an exact Multiple of 3, seek it in the last Line; and divide the Distance between it and the last 1 into 3 equal Parts. And the Division next the last 1 shall be the Root sought.

So

So the Cube Root of 512 is 8: The Cube Root of 125 is 5.

The Multitude of Integral Places may be thus known. If those given do not exceed 3, there is but one Integral Place in the Root. If they do exceed 3 but not exceed 6, there will be 2. If they exceed 6 but not exceed 9, there will be 3, *&c.*

So the Cube Root of 1520⌊875 will be found by the Line to be exactly 11⌊5.

VIII. Three Numbers being given; to find a fourth in a duplicate Proportion.

The Extent from the First to the Second, doubled, will reach from the Third to the Fourth.

Example and Use. The Content of a Circle whose Diameter is 7, is 38⌊5; what is the Content of the Circle if the Diameter be 10⌊5?

Because all similar Superficies are to one another in a duplicate Proportion of any of their Homologous Sides; It is,

As 7 to 10⌊5 Duplicate, so is 38⌊5 to the Content sought. Therefore the Extent from 7 to 10⌊5 Duplicate, will reach from 38⌊5 to 86⌊6, the Content of the Circle sought. For the Extent from 7 to 10⌊5 will reach from 38⌊5 to 57⌊75; and the same Extent will reach from 57⌊75 to 86⌊6 the Content.

In like manner, if the Content of one of any two similar Superficies be given, together with Homologous Lines in each; the Content of the other may be found.

IX. Three

IX. Three Numbers being given; to find a fourth in a subduplicate Proportion.

This is the Converse of the foregoing; therefore half the Distance of the First to the Second, will reach from the Third to the Fourth.

Example and Use. There is a Circle whose Content is 86└6, and its Diameter is 10└5; there is another I would have contain 38└5: I demand its Diameter?

Because the subduplicate Ratio of similar Superficies are as their like Lines: The subduplicate of 86└6 to 38└5, is as 10└5 to the Answer. Therefore bisect the Distance between 86└6 and 38└5, which will be at 57└75; I say the Distance from 57└75 to 385, will reach from 10└5 to 7, the Diameter sought.

X. Three Numbers being given; to find a fourth in a triplicate Proportion.

The Triple of the Extent of the First to the Second, will reach from the Third to the Fourth.

Example and Use. The Weight of an Iron Bullet, whose Diameter is 4 Inches, is 9 *lb.* what will a Bullet weigh whose Diameter is 8 Inches?

Since all similar Solids are to one another, in the triplicate Proportion of their like Lines: The Triplicate of 4 to 8, is as 9 to the Answer. Therefore,

The

The Triple of the Extent from 4 to 8, will reach from 9 to 72 the Weight fought. For the Extent from 4 to 8 will reach from 9 to 18, from 18 to 36, and from 36 to 72.

The like of all other fimilar Solids.

XI. Three Numbers being given; to find a Fourth in a fubtriplicate Proportion.

One third Part of the Extent of the Firft to the Second, will reach from the Third to the Fourth.

Example and Ufe. An Iron Bullet of 4 Inches Diameter weighs 9 *lb.*; what is the Diameter if the Weight be 72 *lb.*? The Subtriplicate of 9 to 72, is as 4 to the Anfwer.

Therefore divide the Diftance between 72 and 9 into 3 equal Parts, and the laft Divifion will fall on 18; then the Diftance from 9 to 18 will reach from 4 to 8.

XII. What hath been faid in this Chapter of Numbers, is applicable to the Lines, Superficies, Bodies, *&c.* that thefe Numbers may repefent. And this too, to a much greater Purpofe than could be done by the Sectoral Lines of *Superficies* and *Solids*. For Inftance, two Lines A, B being given; to find a mean Proportional: Whether you are to work by the Sectoral Lines, or the Line of Numbers; thefe Lines A, B, given, muft be meafured on fome Line of equal Parts. And the Middle between the Points on the Line of Numbers reprefents the Length of the mean Proportional in Numbers. By the Sectoral Lines, the mean Proportional Line itfelf

itself is sought; and therefore it must neither be too long nor too short for your Sector. For if it or the given Lines should be so, you must prepare them or their Parts to fit them for the Work; which is not required on the Numbers. And whatsoever hath been said of finding two mean Proportional Numbers, between two Numbers given, may be applied to two given Lines, if they are measured by some Line of equal Parts.

CHAP.

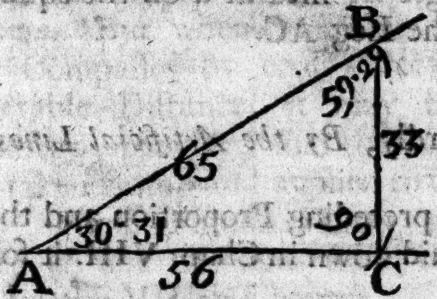
CHAP. XIV.

Of the joint Use of the preceding Lines in the Solution of right lin'd Triangles.

1. *From the Chords and Lines by Protraction.*

2. *From the other Sectoral Lines* } *by Calculation.*
3. *From the Artificial Lines* }

I. IN a right lined Triangle ABC, right lined at C, are given the Hypothenuse AB (65) and the oblique Angles A (30°. 31) B (59. 29); to find the Legs AC, BC.



First, By Protraction.

Draw AC at pleasure; make the Angle A 30° 31′ as taught in the 10th and 11th Chapters; lay 65 equal Parts on AB: At B make the Angle B 59° 29′, or from B let fall BC perpendicular to AC; and so will the Triangle ABC be protracted: Then AC and BC may be measur'd by the same Scale that AB was laid down by.

2dly, *By Calculation from the Sectoral Lines.*

Since the Radius is to the Sine of either Angle (A), as the Hypothenuse (AB) is to (BC) the Leg opposite to that Angle.

* Take AB (65) from the Lines of any equal Parts, and make it a Parallel at the Sine of 90. Then the Parallel at the Sine of (30° 31′) the Angle A, measur'd on the same equal Parts, gives (33) the Leg BC. And the Parallel at (59° 29′) the Degrees of the Angle B, measur'd on the equal Parts, gives 56 the Leg AC.

Thirdly, *By the Artificial Lines.*

By the preceding Proportion and the general Law laid down in Chap. VIII. it follows,

* See Chap. VIII.

That

That the Extent from the Sine of 90 Degrees to the Sine of 30°–31, will reach on the Numbers from 65 to 33.

And the Extent from the Sine of 90 to the Sine of 59°–21 will reach from 65 to 56 on the Numbers.

Or thus, The Extent from the Sine of 90° to the Hypothenuse AB (65) on the Numbers, will reach from the Sine (30°–31′) the Angle A, to BC (33) on the Numbers. And the same will reach from the Sine of (59°–29′) the Angle (B) to (56) AC on the Numbers.

General Observations.

This latter Method requires but one opening of the Compasses, to find both the Legs. It is called cross Work. It is sometimes very convenient, sometimes very improper. When the Extent of the Compasses by cross Work is taken from Points almost over one another, the Result will be very doubtful and imperfect. This Inconvenience may sometimes be remedied by counting on the other 5 Numbers. But when the Points to be taken in the Compasses by cross Work, are at a considerable Distance, it may be practis'd with Safety; provided that these Extents are nicely taken from, and applied to the Fiducial Lines. These Fiducial Lines are known, because all the Divisions stand on them, as hinted in the VIIIth Chapter.

When

When several Proportions are the same, and so may be wrought by cross Work at one opening of the Compasses, as in the Solution of this Example; or if the Distance to be taken off the same Line be too great for the Compasses, use cross Work if it may be remedied thereby.

It was hinted (Chap. VIII.) that the Extent from the 1st to one of the middle Terms, will reach from the other, applied the same way, to the Answer. That is, if you extend from the 1st, upwards, or towards the Right-Hand, the Application of that Extent, to find the 4th, shall be made from the given Quantity upwards. And if the Extent be made downwards, the Application shall be made downwards also.

The Lines of Sines and Tangents begin at a little less than 1 Degree. And in some Examples, the Extent being applied, will fall below the Divisions on the Sines or Tangents. In this Case, keeping the Compasses with the same Extent, bring 1 Foot of the Compasses to 1 Degree, and where the other falls, hold it fast till you have so squeez'd the Legs together, that the other Point which was plac'd at 1 Degree, falls on the Term to which the Application was made. Then this Distance laid from 60 on the Numbers downwards, will reach to the Minutes, and Parts of Minutes requir'd.

Example 1. Let the Proportion be, as 91 to 22, so is the Sine of 2 Degrees to a fourth Sine. So the Extent from 91 to 22, will reach

reach from the Sine of 2 Degrees downwards, beyond the End of the Sector. Therefore with the ſame Extent one Foot placed in one Degree, and the other will reach upwards to ſomewhat more than 4°, where hold the Point faſt; then ſqueeze the Compaſſes together till the other Point falls on 2, the Point to which the Application was made. This Diſtance applied downwards on the Numbers from 60, will reach to 29, which is the Anſwer in Minutes.

Example 2. As 95 is to 19, ſo is the Sine of 1°∟30′ to another Sine. So the Extent from 95 to 19 will reach from the Sine of 1° 30′ below the End of the Sector. This Diſtance applied upwards from one Degree, reaches to above 5, where being held faſt, and the other Foot brought to 1°∟30, you have a Diſtance, which laid on the Numbers, reaches from 60 downwards to 18, for the Minutes ſought.

II. In a right-lin'd Triangle ABC, right-angled at C, there is given the Angles A (30°∟31′) and B (59∟29′) and one Leg AC (56): To find the other Leg and the Hypothenuſe.

First, *By Protraction.*

Draw AC at pleaſure; make the Angle A 30°∟31′; lay 56 equal Parts from A to C, and at C raiſe BC perpendicular to AC; and produce AB, CB till they meet in B; ſo

will the Triangle be constructed: And the Hypothenuse AB and the Leg BC may be measur'd from the Scale AC was laid down by.

Secondly, *By the Sectoral Lines.*

Since as the Sine of the Angle (B) opposite to the given Leg (AC) is to that Leg, so is the Radius to (AB) the Hypothenuse; and so is the Sine of the Angle A to (BC) the Side opposite to it. Therefore,

Open the Sector so that AC (56) on the *Line of Sines*, may be a Parallel at the Sine of (59°∟29′) the Angle B; and then a Parallel at the Sine of (30°∟31′) the Angle A will, when measur'd on the Lines, give (33) the Leg BC; And the Sector remaining with the same Aperture, a Parallel at the Sine of 90, will give AB (65) on the *Line of Sines*.

Thirdly, *By the Artificial Lines.*

From the preceding Proportion and the general Law in Chap. VIII. The Extent from the Sine of 59°∟29′ to the Sine of 90° will reach from 56 on the Numbers to AC 65. And the Extent from the Sine of 59°∟29′ to the Sine of 30°∟31 will reach from 56 on the Numbers to BC 33.

Or, at one opening by the Cross Work thus. The Extent from the Sine of 59°∟29′ to 56 on the Numbers, will reach from the Sine

Sine of 90° to 65 on the Numbers; and from the Sine of 30° 31′ to 33 on the Numbers.

III. In a right-lin'd Triangle ABC, right-angled at C; there is given the Hypothenuse AB (65) and one Leg BC (33): To find the other Leg AC, and the Angles A and B.

First, *By Protraction.*

Draw AC at pleasure, and at C raise the Perpendicular BC, and lay thereon 33 from any Line of equal Parts from C to B. Produce AC towards A. With 65 in your Compasses taken from the same Line of equal Parts, one Foot in B with the other describe an Arch cutting AC in A, and draw AB: So will the Triangle be protracted. And the Leg AC as well as the Angles may be measured.

Secondly, *By the Sectoral Lines.*

Since the Hypothenuse (AB) is to the Radius or Sine of 90°, as the Leg (BC) to the Sine of its opposite Angle A. And since the Radius or Sine of 90° is to the Hypothenuse AB, so is the Sine of the Angle B to its opposite Side AC. Therefore,

Make the Sine of 90 a Parallel at the Hypothenuse AB (65) on the *Line of Lines*, and the Parallel at the Leg BC (33) will be the Sine of (30° 31′), the Angle A. And since (30° 31′ taken from 90, leaves 59° 29) for the Angle B; the Sine of 59° 29′ carried along the *Line*

of Lines will be found to be a Parallel at 56, which is the Leg AC.

But the Leg AC might have been found without finding the Angles, by the *Line of Lines* only. For (by the 11th Article of the 11th Chap.) open the Sector so that the *Line of Lines* may form a right Angle, and count the given Leg BC (33) on one Leg; and with the Hypothenuse AB (65) in the Compasses, from 33 as a Center turn the Compasses about till the other Leg falls on the other *Line of Lines* in the Fiducial Line, which you will find to be at 56, which is AC the Leg sought.

Thirdly, *By the Artificial Lines.*

From the preceding Proportion, the Extent from 65 to 56 on the Numbers will reach from the Sine of 90 Degrees to the Sine of 30°∟31′ the Angle A. And so as before the Angle B will, by subtracting the Angle A (30°∟31′) from 90, be found to be 59°∟29′. And then the Extent from the Sine of 90° to the Sine of 59°∟29′, will reach from 65 on the Numbers, to (56) the Leg AC.

Or, by the Cross Work at one opening of the Compasses. The Extent from 65 on the Numbers to the Sine of 90 Degrees, will reach from 33 on the Numbers to the Sine of (30°∟31′) the Angle A. And as before the Angle B is 59°∟29′: Then the same Extent will reach downwards from the Sine of

of (59° L 29) the Angle B to AC (56) on the Line of Numbers.

IV. In a right-angled right-lined Triangle ABC, there are given the Legs AC (56) and BC (33); to find the Angles A, B, and the Hypothenuſe AB.

First, *By Protraction.*

Draw AC, and perpendicular thereto raiſe BC: Make AC 56, and BC 33 (from any Scale of equal Parts, and draw AB); ſo is the Triangle protracted, and the Hypothenuſe and the ſought Angles may be meaſur'd.

Secondly, *By the Sectoral Lines.*

Since one Leg is to the other as the Radius (here the Tangent of 45°) is to the Tangent of the Angle oppoſite to the other Leg. And then the Hypothenuſe may be found by the ſecond Example. Therefore,

Make the Tangent of 45° a Parallel on the Lines, at (56) the Length of AC one of the Legs; and then the Parallel at 33 on the Numbers meaſur'd on the Tangents, will give (30° L $31'$) the Angle A. Then, as in the ſecond Example, may be found the Hypothenuſe AB.

But the Hypothenuſe might have been found without the Angles by the help of the Lines of Lines only. For open the Sector ſo that the Lines of Lines may form a right Angle; then count AC (56) on one Line,

 and

and BC (33) on the other; and the Distance from 56 to 33 measur'd on the Lines, will be AB (65) the Hypothenuse.

Thirdly, *By the Artificial Lines.*

From the preceding Proportion, the Extent from AC (56) one Leg, to BC (33) the other Leg on the Line of Numbers, will reach from the Tangent of 45° to the Tangent of (30°∟31′) the Angle A opposite to the latter Leg. This Angle being found, the Hypothenuse may be found as in the second Example.

The Cross Work here can never assist you.

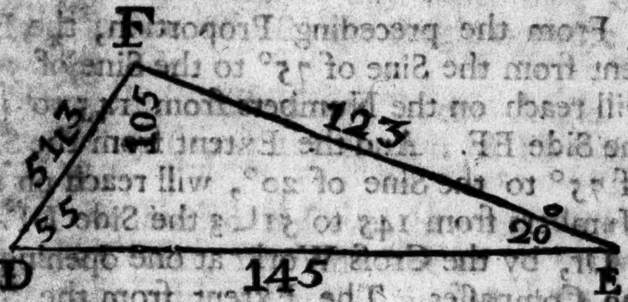
V. In any right-lined Triangle DEF; the Angles D, E, F, being given (55, 20, 105) to find the Ratio of the Sides.

Measure the Sines of those Angles on the *Line of Sines*, and you will have the Proportions of the Sides; *viz.* 81∟9, 34∟2, 96∟5; always observing that the greater Side is opposite to the greater Angle.

This might be perform'd by Protraction or by the Artificial Lines; by assuming any one Length for either of the Sides. But then it would change into the following, which see.

VI. In

VI. In any right-lined Triangle DEF, The Angles D (55) E (20) F (105) and one Side DE 145; to find the other two Sides DF, EF.



First, *By Protraction.*

Draw DE at pleasure, and make it 145 from any Line of equal Parts. At D make an Angle of 55°, and at E one of 20 (*per* Chap. 10.) and produce the Lines till they meet in F. Then is the Triangle protracted, and the sought Lines DF, EF, may be measur'd by the same Scale that DE was protracted by.

Secondly, *By the Sectoral Lines.*

Since the Sine of any Angle (F) is to its opposite Side DE, as the Sine of any other Angle D to its opposite Side.

Make DE (145) a Parallel at the Sine of (105°, that is, by the 3d Observ. in Chap. I. the Sine of 75°) the Angle F; and then the

Parallel at the Sine of (55°) the Angle D, will, when meaſur'd on the Lines of Lines, give (123) the Side FE. And the Parallel at the Sine of (20) the Angle E, will give 51∟3.

Thirdly, *By the Artificial Lines.*

From the preceding Proportion, the Extent from the Sine of 75° to the Sine of 55°, will reach on the Numbers from 145 to 123 the Side EF. And the Extent from the Sine of 75° to the Sine of 20°, will reach on the Numbers from 145 to 51∟3 the Side DF.

Or, by the Croſs Work at one opening of the Compaſſes. The Extent from the Sine of 75° to 145 on the Numbers, will reach from the Sine of 55° to (123) the Side FE; and alſo from the Sine of 20° to (51∟3) the Side DF.

VII. In any right-lined Triangle DEF, two Sides DF (51∟3), FE (123), and the Angle D (55°) oppoſite to one of them, being given; to find the other Angles F, E, and the other Side DE.

First, *By Protraction.*

Draw DE, make the Angle D 55°, and make DF from a Scale of equal Parts 51∟3. Then take FE 123 from the ſame Line of equal Parts; and with one Foot in F deſcribe an Arch cutting the Line DE in E, and draw FE: So you will have protracted the Triangle.

And

And the Angles F, E, and the third Side DE may be meaſur'd.

Secondly, *By the Sectoral Lines.*

Since any Side (FE) is to the Sine of its oppoſite Angle (D), as any other Side DF is to the Sine of (E) the Angle oppoſite to the other Side.

Make the Sine of (55°) the Angle D a Parallel at (123) the Length of FE; and then, &c.

But we are to obſerve that 123 falls ſo near the Center, that the Sine of 55° is too great to be made a Parallel at 123. Therefore make 123 a Parallel at the Sine of 55, and then (51∟3) the Line DF carried along the Sines till it becomes a Parallel, will give us the Sine of (20°) the Angle E. Now becauſe the two Angles D and E are known, therefore the third F is alſo known; for all the Angles of every right-lined Triangle taken together will be equal to two right ones; whoſe Meaſure is 180 Degrees. Therefore the two Angles D and E, 55 and 20, that is, 75 taken from 180, will leave 105 for the Angle F. And the Parallel at the Sine of (105°, that is) 75° gives on the Lines DE 145.

But the third Side DE may be found without the ſought Angles, by the *Line of Lines.* For ſo open the Sector (by the 12th of the 11th Chapter) that the Angle may be 55°; and count on one Leg 51∟3: then with 123 in the Compaſſes from 55, turn the other

other Point about till it falls on the Lines in the other Leg of the Sector, which you will find to be at 145, which is the third Side sought.

Thirdly, *By the Artificial Numbers.*

From the preceding Proportion, the Extent from 123 on the Numbers to 51⌊3, will reach from the Sine of 55° to the Sine of 20° the Angle E. And then as before, the Angle F will be found to be 105; which hath the same Sine that 75 hath. And then the Extent from the Sine of 55° to the Sine of 75°, will reach from 123 on the Numbers, to 145 the Side DE.

Or, by the Cross Work, at one opening of the Compasses. For the Extent from 123 on the Numbers to the Sine of 55° will reach from 51⌊3 on the Numbers to the Sine of 20°. And the same opening will reach the contrary way from the Sine of 75 Degrees, to 145 on the Numbers.

VIII. In any right-lined Triangle DEF. Two Sides DF (51⌊3), FE (123), and the Angle F (105°) contain'd by those Sides being given; to find the Angles D and E, and the third Side DE.

First, *By Protraction.*

Draw DF at pleasure, and make the Angle F 105°. Then from some Scale of equal Parts make FD 51⌊3 and FE 123; and draw DE: So is the Triangle protracted: And

And the Angles D, E, and the Side DE may be measur'd.

Secondly, Since the Sum of the given Sides is to their Difference, as the Tangent of half the Sum of their opposite Angles is to the Tangent of half their Difference.

And in this Example the Angle F being 105°, and so the Sum of the Angles at D and E 75°, and the half Sum 37°∟30′.

Make the Sum of the Sides 174∟3 a Parallel at the Tangent of 37°∟30′. And then 71∟6, the Difference of the Sides carried along the Tangents, will be a Parallel at 17°∟30′, which is the half Difference. But the half Difference (17°∟30′) added to the half Sum (37°∟30′) gives (55°) the greater Angle (D). And the half Difference (17°∟30′) taken from the half Sum (37°∟30′) leaves (20°) the lesser Angle E. And now by the 6th Case, the third Side DE may be found. But you may observe, that when the half Sum of the opposite Angles is more than 45 Degrees; and the half Difference proves less than 45 Degrees, it will be troublesome to perform it by the Sectoral Lines. But this Inconvenience is very well cured by help of the Artificial ones.

But the third Side may be found without the unknown Angles very easily by the *Line of Lines*; and then the Angles may be found, as in the following.

For (by the 12th of Chap. XI.) open the Sector till the Lines form an Angle of 105°; count 65 on the Lines on one Leg, and 123

on

on the other, and the Diſtance from this 65 to 123 is 145 the third Side.

Thirdly, *By the Artificial Lines.*

From the preceding Proportion, the Extent from 174∟3 on the Numbers to 71∟6, will reach from the Tangent of (37°∟30′) the half Sum of the unknown Angles, to the Tangent of (17°∟30′) their half Difference as before. And ſo as before the Angle D is 55°, and E 20°. The Third now is found by the laſt.

The Croſs Work cannot here do Service.

If the Sum of the unknown Angles is more than 45 Degrees, and their Difference proves to be fewer, then the Extent from the Sum of the Sides to the Difference of the Sides, will reach from the half Sum of the unknown Angles upwards beyond the Tangent of 45 Degrees. In which Caſe, the Compaſſes keeping the ſame Opening, bring the upper Point back to 45, then keep the other where it reſts, and bring the Point which was plac'd at 45, to the Tangent of the half Sum. Now lay this laſt Extent from 45 downwards, and you will have the Tangent of the Degrees ſought: *e. g.* Let it be as 90 to 30, ſo the Tangent of 55° to another Tangent. Then the Extent from 90 on the Numbers to 30, will reach from the Tangent upwards beyond 45. But this Diſtance being laid from 45 downwards, keep the lower Point fix'd, and bring the Point to (the Tangent of 55°) the Place

from

from whence the Application of the Extent was to be made; and this last Distance will reach from 45° downwards, to the Tangent of 25° L 28′, the Answer.

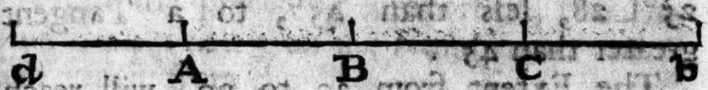
In like manner, if it had been as 30 to 90, (or any other Proportion) so the Tangent of 25° L 28, less than 45°, to a Tangent greater than 45°.

The Extent from 30 to 90, will reach from the Tangent of 25° 28′ beyond 45°, that is, to a Tangent greater than 45°: With this opening bring one Point to 45; where this falls hold it fast, and bring the other to 25° 28′: Apply this last Extent from 45, and it will reach in the upper or inverted Tangents, to the Tangent of 45, the Answer.

The Reason of this Operation is this: The upper Line of Tangents should have been conceiv'd as continued beyond 45 right forwards. But because the Divisions would have fallen beyond 45 upwards, in the same manner that they fall under 45 the contrary way; to avoid Incumbrance and Charge, the same Line is number'd backwards, but with the upper Number. But from hence it appears, that the Figures above 45 increase towards the Left-Hand, and so they must be accounted upwards from the Right to the Left, and downwards from the Left to the Right.

Again, in our last Example the Extent from 30 to 90 reaches from 25° L 28′ beyond 45. Let (in Fig. following) 25° 28′ be denoted by A, 45 by C, and the Extent by A *b*. Now it's evident that if the Tangents had

had been continued beyond 45, *b* would have given the Anſwer. But the Tangents are not continued beyond 45, for inſtead thereof they are *folded backwards; conſequently the Point where the Anſwer will be found,



is as far below C as *b* is above it. Therefore C*b* laid from C to B gives at B 55°, the Anſwer. But becauſe I cannot conveniently meaſure C*b*, *b* falling off the Rule, I lay A*b* from C to *d*, and meaſure *d* A which is equal to C *b*, and lay it from C to B; and ſo gain the Anſwer. I ſaid A *d* was equal to C *b*, which will thus appear: By the Operation C*d* was made from the ſame Extent that A *b* was, whence C *d* is equal to A *b*; and by taking away the common Part AC, there remains A *d* equal to C *b*. Therefore A *d* may be uſed inſtead of C *b*.

It may be alſo obſerv'd, that if the Extent taken from the Numbers be greater than that from 10 to 1, and perhaps too long for the Compaſſes; and this is to be applied from the Tangent of 45° upwards or downwards, or the Sine of 90°, that then the Diſtance from 10 to 1 may be neglected; and the remaining Diſtance applied from 5° ∟ 43′ on the Tangents, and from 5° ∟ 45′ on the Sines towards the Left-Hand, will give the Tangent or Sine ſought: *e. g.* Let 90 be to 8 as the Sine of 90° to another Sine: Then

*

Then because the Extent from 90 to 9, is equal to the Extent from 10 to 1; I take the Extent from 9 to 8, or, which is the same Thing, I seek both Numbers on one Line, and then the Extent will reach from the Sine of 5°└45′ to the Sine of 5° 07′, or on the Tangents from 5° 43′ to 5°└05.

In like manner, if you are to take the Extent from 90° on the Sines, or 45° on the Tangents, to a Sine below 5°└45′, or a Tangent below 5°└43′; extend the Compasses from 5°└45′ or 5°└43′ to the Sine or Tangent given; and this Extent will reach from the third Term, counted on the upper Line of Numbers, downwards to the 4th sought, to be esteemed as tho' it were in a Line of Numbers below where it falls.

If the Proportion had been from a Sine below 5° 45′ to the Radius; take the Extent from the given Sine to 5°└45′, and it will reach from the third Term counted on the lower Numbers upwards to the 4th sought, to be esteem'd as on a Line of Numbers next above it.

And if it were from the Sine of 90 to a Sine of Degrees, a few more than 5°└45, as 6°└00, and the Compasses too short; then the Extent from 5°└45′ to 6°└00′, will reach from the third Term counted on the lower Line of Numbers upwards to the 4th sought, to be esteem'd as on a Line of Numbers next below where it falls.

If it be as a Sine of 6°└00 to the Sine of 90°, so a Number to another sought. It may be as 6°└00′ to 5°└45′, so is the third

on

on the upper Numbers downwards to the 4th ſought, to be eſteem'd as on a Line of Numbers next above where it falls.

If the Extent on the Numbers be almoſt the Length of one of the Lines of Numbers, and be afterwards to be apply'd to the Sine of 90°: Find both Numbers on one Line, and the Extent will reach from 5°∟45′ on the Sines, to the Sine ſought.

Example 1. As the Sine of 90° to the Sine of 3°; ſo is 57 to 2∟98.

As the Sine of 4° to the Sine of 90°; ſo is 8∟5 to 122.

As the Sine of 90° to the Sine of 6°; ſo is 59 to 6∟13.

As the Sine of 6° to the Sine of 90°; ſo is 63 to 605.

As 57∟5 is to 6∟3; ſo is the Sine of 90° to the Sine of 6°∟03′.

What hath been ſaid in theſe Obſervations of the upper and lower Tangents, when both are in uſe together, may be applied to a Sine and a Secant when uſed together.

And what hath been ſaid of comparing the Sine of 90 Degrees with a Sine leſs than, or very near to the Sine of 5°∟45′; and of comparing the Tangent of 45° with a Tangent leſs, or little more than the Tangent of 5°∟43′, may be applied to a Tangent greater or little leſs than the Tangent of 84°∟17′, changing the Word upwards for downwards. Underſtand the ſame of the Secant of 84°∟15′.

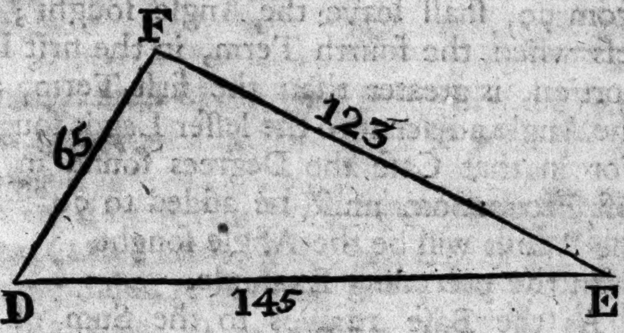
IX. The

IX. The three Sides of a right-lined Triangle being given, to find either of the Angles.

Let the three Sides be DE 145, DF 65, FE 123: To find the Angle F.

First, *By Protraction.*

Draw DE at Pleasure, and make it 145 equal Parts. Take DF 65, and FE 123 of those equal Parts in your Compasses, and making D and E Centers, describe Arches, cutting one another in F. Draw DF, EF, so is the Triangle protracted, and the Angle F may be measured as taught in the 10th and 11th Chapters.



But the Triangle may be represented on the Legs of the Sector; for count DF 65 on one of the *Lines of Lines*, and 123 on the other; take 145 in the Compasses, and open the Sector 'till this Extent reaches from 65 to 123; and then the Angle that the *Lines of*

Lines make, is the Angle F, which may be meaſured by the Precepts of Chap. XI.

And the Angle may be found by the two following Proportions. As either of the Sides adjacent to the ſought Angle (which call the *Baſe*) is to the Sum of the other two Sides (which call *Legs*); ſo is the Difference of thoſe two, to a fourth.

Then add and ſubtract the firſt and laſt Terms in the preceding Proportion; and it follows, That if the Angle ſought be adjacent to the $\left\{\begin{matrix}\text{leſſer}\\\text{greater}\end{matrix}\right\}$ Leg, then the $\left\{\begin{matrix}\text{leſſer}\\\text{greater}\end{matrix}\right\}$ Leg, is to the Radius, or Sine of 90°, as the Half $\left\{\begin{matrix}\text{Difference}\\\text{Sum}\end{matrix}\right\}$ of the firſt and laſt Terms in the preceding Proportion, to the Sine of a Number of Degrees, which taken from 90, ſhall leave the Angle ſought; unleſs when the fourth Term, in the firſt Proportion, is greater than the firſt Term, and the Angle adjacent to the leſſer Leg is ſought: For in that Caſe the Degrees found in the laſt Proportion, muſt be added to 90°, and the Reſult will be the Angle ſought.

In the preceding Example,

As the Baſe 123, is to the Sum of the Legs 210; ſo is their Difference 80, to a fourth 136 $\frac{1}{2}$.

Therefore the Sum of the firſt and laſt Terms, is 259⌊5, and their Difference is 13⌊5. Whence the half Sum is 129⌊75, and the half Difference 6⌊75.

And becauſe the Angle ſought is adjacent to D F, the leſſer Leg, it is, As D F (65) is to

to the Radius; so is the half Diff. (61.75) of the first and last Terms in the preceding Proportion, to the Sine of 6°; which, because the fourth Term in the first Proportion, is greater than the first, and the Angle adjacent to the lesser Leg is sought; add it to 90°, and you have 96°, the Angle sought.

But, by the Help of the *Line of Numbers*, and the artificial versed Sines running close to the right Sines, the sought Angle may be found much easier: Thus,

The Sides containing the sought Angle F - - -	D F	65
	E F	123
The Side opposite to the sought Angle - - - -	D E	145
The Sum of all the Sides - - -		333
Half the Sum of all the Sides - -		$166\frac{1}{2}$
		145
Half the Sum of the Sides wanting the Opposite - - -		$21\frac{1}{2}$

The Extent on the Numbers from the half Sum ($166\frac{1}{2}$) to (E F 123) one of the containing Sides, will reach from the other containing Side, to a fourth; where hold the Point unmov'd, and extend the other Point to ($21\frac{1}{2}$) half the Sum of the Sides lessened by the opposite Side: And this second Extent will reach from the Beginning of the versed Sines to (96°) the Angle F required.

As Trigonometry is an Art uſeful in almoſt all the practical Parts of Mathematicks, and ſince, by the Sector, the various Caſes (as evidently appears from the Examples here laid down) are conciſely and elegantly ſolved, I have been more particularly full to exemplify and explain the Uſe of the Sector herein.

I believe alſo, by this Time the Reader does perceive, that the Sector, as it is now made, is capable of performing all the more uſeful Problems required in common Life, with leſs Confuſion, more Speed and Exactneſs, than by the old Sectors, loaded with Multitudes of Lines: And that the artificial Lines, not dreamt of before the great Invention of Logarithms, are fitter for theſe Operations, in all Caſes where proportional Numbers are concerned, than any other Lines that have been laid down on Inſtruments; and therefore were, with very great Judgment, cauſed to be cut on the Sector: And this will again appear in the Application of theſe artificial Lines, to Problems in Geography, Navigation, Dialling, Aſtronomy, &c.

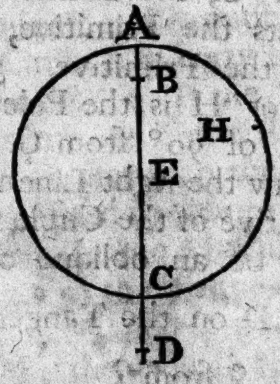
CHAP.

CHAP. XV.

The joint Use of the Chords, and the natural Tangents and Secants, in protracting and measuring the Representatives of spherical Arches and Angles, on a Plane.

IN the Management hereof, I beg leave, for Brevity's ſake, to call theſe plain Repreſentatives, the ſpherical Angles, Arches, and their Poles, themſelves. Every where the Radius is the Radius of the Primitive Circle, unleſs otherwiſe expreſſed.

I. A Point being given: To find another diametrically oppoſite to it.



Thro' the given Point, and the Center of the primitive Circle, draw a right Line. Then if the given Point be A on the primitive

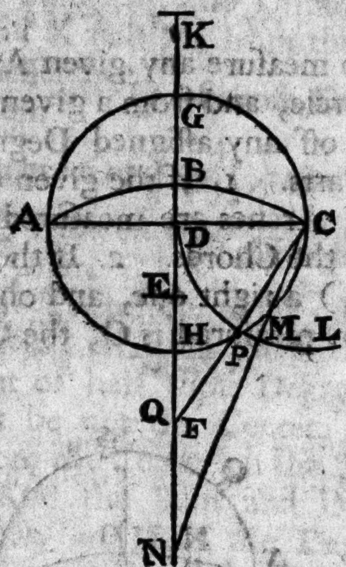
Circle, the Point sought will be C also on the primitive Circle. But if the given Point be B, not on, but in the primitive Circle, the Sector being opened to the Radius E A, measure E B on the Tangents: Then lay the Tangent of the Complement of E B, from E to D, and then D is the Point.

II. To draw a great Circle through H, B, two given Points. By the preceding find a Point (D) diametrically opposite to (B) either of those given; and then thro' the three Points A, B, C, describe the Periphery of a Circle, and it is done.

III. A great Circle being given, to find its Poles: And a Pole being given, to describe the great Circle. 1. If the Circle given be the Primitive, its Center is its Pole. 2. If the Circle be a right one, as A D C, its Poles are found by laying of the Chord of 90° from A or C, the Points where the right Circle meets the Primitive, to the Points H and G on the Primitive. 3. Therefore, if a Point G or H is the Pole of a Circle, lay the Chord of 90° from G or H, to A and C, and draw the right Line A D C; it is the Representative of the Circle required. 4. If the Circle be an oblique one, as A B C; measure B D on the Tangents $\left\{\begin{matrix}\text{subtract}\\ \text{add}\end{matrix}\right\}$ the Degrees $\left\{\begin{matrix}\text{from}\\ \text{to}\end{matrix}\right\}$ 45, and lay the Tangent of the $\left\{\begin{matrix}\text{Remainder}\\ \text{Sum}\end{matrix}\right\}$ from D to $\left\{\begin{matrix}\text{E}\\ \text{K}\end{matrix}\right\}$

so

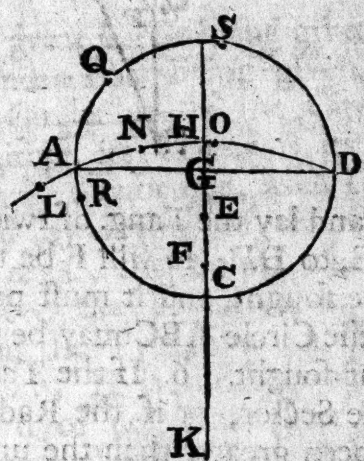
ſo will E, K be the Poles ſought. 5. If the Pole E be given to deſcribe the Circle, draw D E, and at right Angles to it, through the Center, draw A D C: Meaſure D E on the



Tangents, and lay the Tang. of twice the Degrees of DE, to DF; ſo will F be the Center of the Circle ſought, and it muſt paſs thro' C and A: ſo the Circle ABC may be deſcribed, which is that ſought. 6. If the Tangent had fallen off the Sector, or if the Radius of the Circle had been greater than the upper Tangents could be opened to; from C, with the Radius C D, deſcribe the Arch D L, and lay on it the Degrees of the Tangent from D to M; and draw C M, and produce it 'till it meets D G in N; then will D N be the Tangent, and C N the Secant of D M. 7. If the Pole given had been (K) out of the

Primitive: Then by *n.* 1. find E, another Point diametrically opposite to it, which is the other Pole to this Circle; and then the Circle may be described, as in the 5th of this III.

IV. To measure any given Arch; and on a given Circle, and from a given Point therein, to lay off any assigned Degrees, towards assigned Parts. 1. If the given Circle be the Primitive, Arches are measured and Degrees laid off by the Chords. 2. If the given Circle be (G K) a right one, and one End of the Arch to be measured is G, the Center of the



Primitive; then G H and G K measured on the Tangents (with the Radius of the Primitive) will give Half the Degrees of G H and G K, and these doubled the Degrees of G H and G K: But neither of the Extremities were the Center of the Primitive (as of the

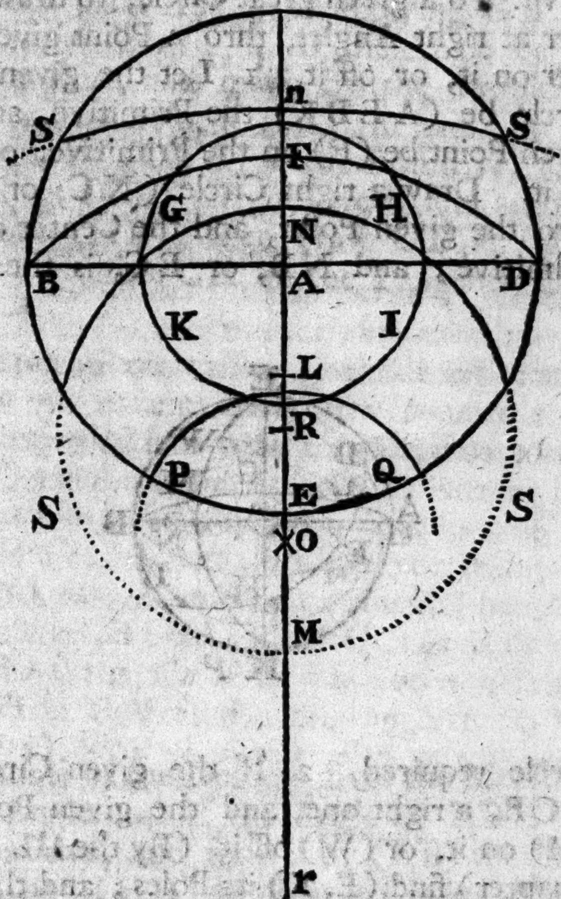
Arches

Arches $\left\{\begin{matrix}\text{F K}\\ \text{F H}\end{matrix}\right\}$ then when the Ends are on $\left\{\begin{matrix}\text{the ſame Side}\\ \text{contrary Sides}\end{matrix}\right\}$ of the Center; then meaſure on the Tangents, the Diſtances of thoſe Ends, F and K and H, from the Center G, and the $\left\{\begin{matrix}\text{Difference}\\ \text{Sum}\end{matrix}\right\}$ of thoſe Degrees, give Half the Degrees of $\left\{\begin{matrix}\text{F K}\\ \text{F H}\end{matrix}\right\}$ and ſo the Double of the $\left\{\begin{matrix}\text{Difference}\\ \text{Sum}\end{matrix}\right\}$ gives the Degrees requir'd. 3. Therefore, to lay off any Number of Degrees: If from the Center, lay off the Tangent of half thoſe Degrees. If the given Point be not the Center, meaſure its Diſtance from the Center on the Tangents in Degrees: And the Sum and Difference of theſe Degrees, and half the Degrees to be laid off, are to be laid off from the Center by the Tangents, towards the given Point, if the Part F K, to be laid off, be requir'd contraryways to the Center; or if F E, the Part to be laid off, be leſs than F G, the Diſtance of F from the Center. But, on the other Side of the Center, if F G, the Part to be laid off towards the Center, be greater than the Diſtance of the given Point F, from G the Center. If half the Degrees to be laid off from F, be $\left\{\begin{matrix}\text{more}\\ \text{fewer}\end{matrix}\right\}$ than the Degrees of the Tangent F G; then F G is $\left\{\begin{matrix}\text{leſs}\\ \text{greater}\end{matrix}\right\}$ than the Arch to be laid off. 5. To meaſure (N L, or N O) Arches of an oblique Circle, or

or of any Circle in general: Find its Pole F within the Primitive, and lay a Ruler from F (to N, L, O) the Extremities of the Arches, and it will meet the Primitive in the Points R, Q, S; then will R Q, and Q S on the Chords, measure L N and N O. 6. On A D, or any Circle whatsoever, from a given Point N, to lay off any assigned Number of Degrees, suppose 56. First find F, its Pole, lay a Ruler from F to N, it will meet the Primitive in Q; then by the Chords lay 56 from Q to R, and from Q to S; and lay a Ruler from F to R and S, it will cross A D in L and O; and so will N L and N O be each 56°, as requir'd.

V. To describe a lesser Circle, parallel to any great Circle given, and at any assigned Distance from its Pole. 1. If the great Circle be the Primitive (and the lesser Circle is at 56°, or any other assigned Distance from its Pole) take the Tangent of (28) Half the assigned Distance, and from A, the Center of the Primitive, describe G H I K, and it is the Parallel required. 2. If the great Circle be (B D) a right one, whose Pole is E, from whence the lesser Circle is to be (30°, or) any assigned Distance, lay from the Chords (30°) the assigned Distance from (E) the Pole, to P and Q. Then with the Tangent of P E, or Q E, describe Arches cutting one another in O; (or the Point O may be found by laying the Secant of 30 from A) and from O, with the same Distance, describe the Circle P Q, and it will be that required. 3. If the

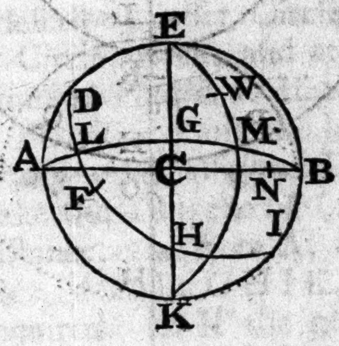
the great Circle be (B F D) an oblique one, let it be required to describe a lesser Circle parallel to it; and at (70°) a given Distance. From L, its Pole, lay (by the IV. of this Chapter) 70° the given Distance both Ways,



on E F from L, *viz.* from L to M, and from L to N. Then bisect M N in R; and from R, with the Distance R M, or R N, describe the Circle M S N, and it is the lesser Circle

Circle required. In like manner the Circle S n S is described parallel to B F D, at 105° Distance from that Pole L; for L n, L m, are each made 105°, and from r, the Middle between n and m, S n S is described.

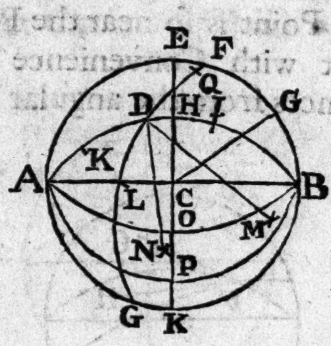
VI. To a given great Circle, to draw another at right Angles, thro' a Point given, either on it, or off it. 1. Let the given great Circle be (A E B K) the Primitive, and the given Point be (E) on the Primitive, or (N) off it. Draw a right Circle (N C, or E C) thro' the given Point, and the Center of the Primitive; and N B, or E C, is the great



Circle required. 2. If the given Circle be A C B, a right one, and the given Point be (M) on it, or (W) off it. (By the III. of this Chapter) find (E, K) its Poles; and through those Poles, and the given Point, draw the Circumference of a Circle, and it is that required. 3. If the Circle be (A G B) an oblique one, and the given Point be (L) on it, or (F) off it: Find (H) the Pole of (A G B) the

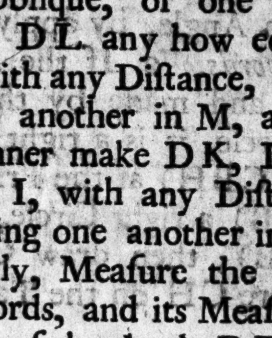
the given Circle. Then (by the II. of this Chapter) deſcribe (DHI) a great Circle thro' (H) the Pole, and (F, or L) the given Point, and it will be the Circle required.

VII. To meaſure any Angle. 1. If the Angle be (GCB) at the Center, it is meaſured as a plain Angle, by applying GB to the *Line of Chords*. 2. If the Angle be at the Primitive, and made by the Primitive and a right Circle, it is a right one. 3. If the Angle be (CBH) at the Primitive, and formed by (CB) a right Circle, and an oblique one (HB); then draw CE at right Angles; meaſure CH on the Tangents, and



the Double of the Degrees gives the Meaſure of the Angle. 4. If the Angle be EBH at the Primitive, and form'd by (EB) the Primitive, and (HB) an oblique Circle. Meaſure CH on the Tang. and ſubtract the Deg. from 45, the then Double of the Remainder gives the Degrees of the Angle (HBE) ſought; and ſo the Sum of the Degrees of the

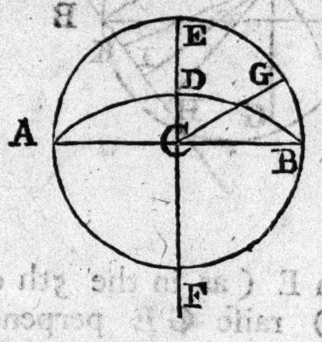
the Tangents H C, C O, doubled, is the Meaſure of the Angle H B O; and the Difference of the Degrees of the Tangents C O, C P, gives half the Meaſure of the Angle O B P. 5. If the Angle be (F D B) neither at the Center, nor at the Primitive, whether both are oblique, or one a right Circle. Make D Q, D L any how equal; and from L and Q, with any Diſtance, deſcribe Arches cutting one another in M, and draw D M. In like Manner make D K, D I, equal, and from D and I, with any Diſtances, deſcribe Arches cutting one another in N, and draw D N. Laſtly, Meaſure the Angle M D N by the Chords, and its Meaſure will be alſo the Meaſure of the Angle B D F. 6. When the angular Point is ſo near the Primitive that you cannot with Convenience lay off the equal Diſtances from the angular Point: Then



draw from (F) the angular Point (F G, F H) Tangents to the Circles, forming the Angle; and by the Chords meaſure the right-lined Angle G F H, and you have the Meaſure of the

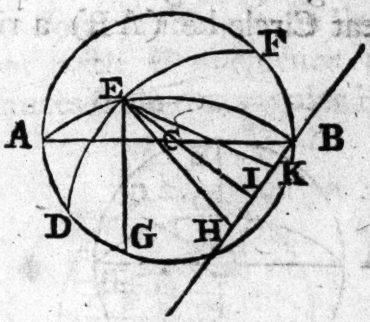
the Angle formed by the Circles. To draw a Tangent to a Circle, without finding the Center, suppose at A; take A B, B C, any how equal, draw A C, and from A describe the Arch D B E; lastly, make B E equal to B D, and draw A E, and it is a Tangent, as required. 7. Universally find the Poles (I, K) of those Circles, and lay a Ruler from the angular Point (F) to those Poles, it will meet the Primitive in L and M, and L M measured on the Chords, gives the Measure of the Angle.

VIII. With a given great Circle, and at a Point therein, to draw another that shall make any assigned Angle with that given. 1. If the great Circle be (A B) a right one,



and the Point given in it be (C) the Center of the Primitive, the Angle (B C G) will be made as a plain one. 2. If the great Circle be the Primitive, and the Point given in it be B, to make an Angle of (40) a given Number of Degrees, lay the Tangent of it from

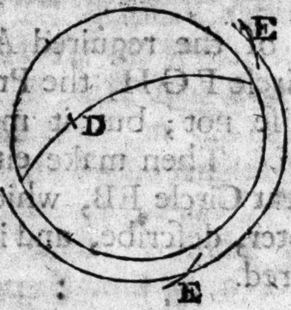
from C to F, or lay the Secant from B to F; so shall F be the Center, and B a Point thro' which the Circle B D A is to be described; and then the Angle D B E will be that required. As in the upper Tangents the Radius C B was fitted to its Radius 45°; so in the Secants, the Radius C B must be fitted to the Beginning of the Secants. 3. If the Angle had been to be made with the right Circle C B, the Tangent Complement of the required Angle C B D, must have been taken to find the Center. 4. If the given Point be neither at the Center, nor at the Primitive, (as E) whether the given Circle be right, or



oblique; from E (as in the 5th of VII. of this Chapter) raise G E perpendicular to (E B) the given Circle: And by the Chords make the Angle G E H equal to the Angle required to be made; draw from E a right Line E C I thro' the Center; measure E C on the Tangents, and lay the Tangent of the Complement of its Double, from C to I, and draw I H perpendicular to C I; so shall H

H be the Center of the Circle D E F, which deſcribed thro' E, will form an Angle as required. And if it had been required to make another Angle of a Magnitude different from the former: Make a plain Angle GEK equal to the required Angle, and in K, where it meets with H I, is the Center of the Circle required. For all the great Circles which paſs thro' the Point E, have their Centers in the Line I H.

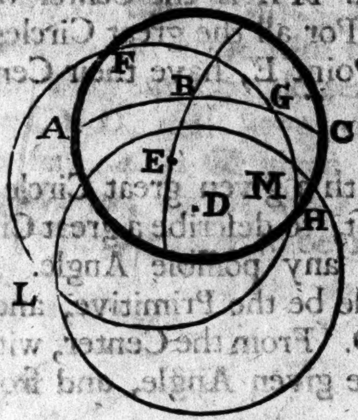
IX. With a given great Circle, and from a Point of it, to deſcribe a great Circle, which ſhall form any poſſible Angle. 1ſt. If the given Circle be the Primitive, and the Point given be D. From the Center, with the Tangent of the given Angle, and from D, with



the Secant, deſcribe Arches cutting one another in (E) the Center of the Circle required, which, becauſe it is to paſs through D, may be deſcribed. 2. If the given Circle *(See Figure of the next Page)* be not the Primitive, let it be A B C, whoſe Pole is D, and let the given Point be E. Then E is

the

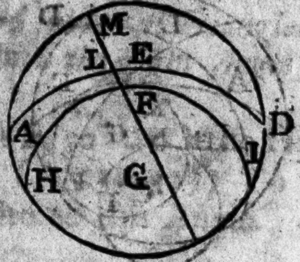
the Pole of ſome Circle which may be deſcribed by the III. of this Chapter, which ſuppoſe to be F G H. From D, the Pole of the given Circle A B C, deſcribe a leſſer Circle at the Diſtance of the Meaſure, or the



Supplement of the required Angle. If this meets the Circle F G H, the Problem is poſſible, otherwiſe not; but it meets it in two Points L, M. Then make either of them a Pole of a great Circle EB, which, by the III. of this Chapter, deſcribe, and it will form the Angle required.

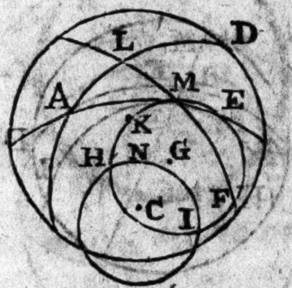
X. With two given great Circles to deſcribe a third that ſhall make Angles with them equal to any aſſigned Angles poſſible, and both on the ſame Side of the required Circle. 1. Let one of the aſſigned Angles be a right one, and the other an oblique one. Find G, the Pole of (AE) one of the given great Circles

cles (by the III. of this Chapter). By the V. describe HFI, a lesser Circle, at a Distance from the Pole (G) equal to the Measure of the Angle to be made, or its Supplement with (A E) that Circle: If this lesser Circle meets (A D) the other given great Circle, the Problem is possible, otherwise not: But



it meets it in two Points (H and I). Then make either of these a Pole of a great Circle, which (by the III. of this Chapter) describe. If you take H for its Pole, it will be the right Circle ILM; the Angle at M will be a right one, and that at L equal to that required. 2. If both Angles are oblique, (*See Fig. of the next Page*) find G, the Pole of A D, one of them (by the III. of this Chapter) and at a Distance from it, equal to the Angle that required Circle is to make, with that (A D) describe (by the V. of this Chapter) a lesser Circle (K H F). Then at a Distance from (C) the Pole of (A E) the other Circle, equal to the Supplement to 180° of the Angle to be made by the required Circle, describe (by the V. of this Chapter) a lesser Circle (H N I). If this

meets the former (KHF), the Problem is possible, otherwise not; but it meets it in two Points (H and I). Then make either of them a Pole of a Circle, which (by the III. of this Chapter) describe. If you take H for the Pole, the Circle will be LM, and it will make the Angles at L and M as requir'd.



CHAP. XVI.

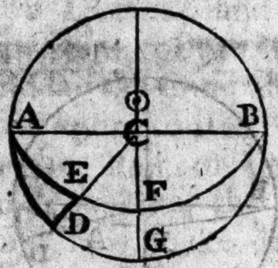
The Use of the SECTOR *in* Spherical Trigonometry, *both by Protraction and Calculation.*

IN the Management of this Chapter, I shall shew the Protraction of all the various Datas from the natural Lines. Give the Solutions to every Case, and the Operations of some, particularly of the more difficult; and leave the rest for the Exercise of Beginners. I farther hint, that

that when Tangents are concerned, it is impossible to work every Example by the natural Lines, without an Expence of a great deal of Time; and then it will scarce be done to the Satisfaction of the Quærist: But the artificial Tangents being always ready, and fit for every Operation, I shall use them for the most Part, and the natural sparingly.

Of Right-angled spherical Triangles. In every Case of which the right Angle is at D.

I. Given (A E 50° 0′) the Hypothenuse, and an Angle (A 23 30): To find (D E) the Leg opposite to this Angle.



First, *By Protraction.*

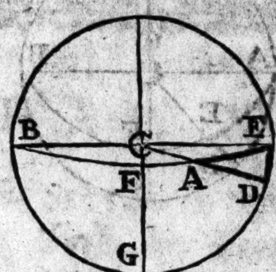
Make (by the VIII. of the last Chap.) the Angle FAG of 23° 30′. And, by the IV. of the last, AE 50°, and draw the right Circle CD; then the Angle at D is a right one, and the Triangle is protracted. Therefore the other Parts A D, E D, and the Angle E, may be measured by the IV. and VII. of the last.

last. The Proportion to find D E is, As the Radius is to the Sine of (A E) the Hypothenuse; so is the Sine of the Angle given (A) to the Sine of (D E) its opposite Side. Therefore, by the natural, or sectoral Sines, make the Sine of 23° 30′, a Parallel at the Sine of 90°, and the Parallel at the Sine of 50°, is the Sine of (17° 47′) the Answer.

Secondly, *By the Artificial Lines.*

The Extent from 90°, to 23° 30′, will reach from 50°, to the Answer 17° 47′.

II. Given (E D 17° 45′) one Leg, and (E 70° 0′) the Angle adjacent: To find (A D) the Leg opposite to it.



First, *By Protraction.*

Make (by the VIII. of the last) the Angle F E G 70° 0′; and by the Chords make D E 17° 47′. Thro' D (by the VI. of the last) draw a great Circle perpendicular to D E; it will be the right Circle C D. So the Tri-

 angle

angle A D E, right-angled at D, will be constructed as required; and now the Hypothenuse A E, the Leg A D, and the Angle A, may be measured (by the IV. and VII. of the last Chapter). The Canon to find A D is, As the Radius is to the Sine of E D (17° 47′); so is Tangent of (70°) the Angle E, to the Tangent of (40° 2′) the Leg sought A D.

2dly, It is worth while to observe, that if you would solve this Question by the sectoral Sines and Tangents on the modern Sector, or on Gunter's, as he made it, you are thus to proceed; Open the Sector to the full Extent, and on the Tangents take the Distance from 70° to 70°: Make this a Parallel at the Sine of 90 Degrees, and then take the Parallel at the Sine of 17° 47′ in your Compasses, and lay the Compasses by: Again, open the Sector quite to its greater Opening, and with another Pair of Compasses, take on the upper Tangents, the Distance from 45 to 45, and now make it a Parallel at 45 on the lower Tangents: Lastly, Take the first Compasses, having the Opening you last gave them, and draw them along the lower Tangents 'till they become a Parallel, which will be at (40°) the Answer. Nor is the Matter any whit the better, if with the former Sectorists you so vary the Proportion, that the Co-Tangents are used instead of the Tangents; and this will always happen when two Tangents are to be compared with one another, whereof the one is greater than of

 45°,

45°, the other of less. Nor is it mended by the Addition of *Gunter*'s natural Tangent on the Edge of the Sector. Lastly, Neither by *Gunter*'s, *Foster*'s, *Collins*'s, &c. can the Solution be wrought out at all, if the Number falls beyond the Tangent on the Sector, without constructing a Tangent, *de novo.* And the same may be said of comparing a Sine with a Secant.

Thirdly, *By the Artificial Lines.*

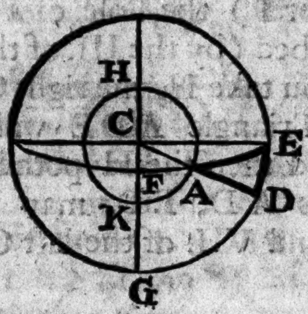
The Extent from the Radius to the Sine of 17° ∟ 47′, will reach from the Tangent of 70, upwards beyond the Tangent of 45; therefore (see the Remarks in the 8th Article of Chap. XIV.) I bring the upper Point to 45, and the other will reach below 70, where hold it fast, and bring the other Point to 70; this Opening laid from 45, will reach to the Tangent of 40°, the Answer. I think this one Example may satisfy the curious Enquirer why the Sector was altered, and the artificial Lines introduced.

III. Given (A D 40°) one Leg, and its opposite Angle (70°): To find the other Angle A.

First, *By Protraction.*

Draw (by the VIII. of the last) F E, so that it makes the Angle FEG 70°; and since AD is to be 40°, CA will be 50°; therefore at the Distance of 50° (by the V. of the last) describe

describe H A K, a leſſer Circle, cutting F E in A. Laſtly, thro' A (by the VI. of the laſt) draw A D perpendicular to E G; but this will be a right Circle, and the Triangle A D E will be conſtructed. And then (by the IV. and VII. of the laſt Chapter) the other Angle A, the other Leg E D, and the Hypothenuſe A E, may be meaſured, and ſo known.



Secondly, *By the Artificial Line*

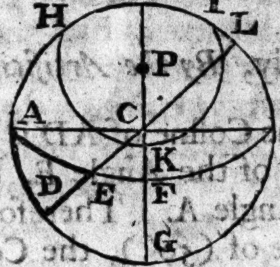
Becauſe the Coſine of AD is to the Radius, as the Coſine of the Angle E, to the Sine of the ſought Angle A. Therefore the Extent from the Sine of (50°), the Complement of A D (40°), to the Radius, will reach from the Sine of (20°), the Complement of the Angle E, to the Sine of (26° 31′) the Angle A ſought.

IV. Given both the oblique Angles (A 48°, E 56°): To find the Hypothenuſe.

First,

First, By Protraction.

Make (by the VIII. of the last Chap.) the Angle F A G 48°; and then (by the IX. of the last Chap.) draw a Circle, so that it may make an Angle with A F of 50°, and with A G, a right Angle; that is, at the Distance of 56° from P, the Pole of A F, describe a lesser Circle H K I, which meets the primitive Circle A G in the Points H and I; then make H (or I) the Pole of a great Circle, which described (by the III. of the last Chap.) will be, if you take H, the right Circle L C D; and so the Triangle A D E will be that required. And then the Hypothenuse (A E), and the Legs A D, E D, may be measured by the IV. and VII. of the last Chapter.



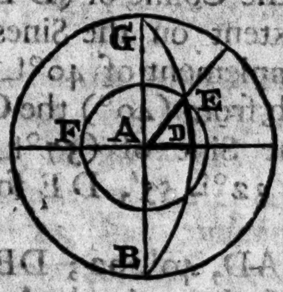
Secondly, By the Artificial Lines.

Because the Tangent of (E 56) one of the Angles, is to Tangent of (52) the Complement of (48) the other Angle, as the Radius is to the Sine of the Complement of (A E) the Hypothenuse. In order to get in your

Compasses

Compasses the Distance from the Tangent of 56°, which is above 45°, to the Tangent of 42°, which is less than 45°; take the Distance from the Tangent of 56°, one of the two, to the Radius, and lay it from 42°, the other, downwards; then take the Extent from the lower Point of the Compasses to 45°, and it is that Distance sought which will reach from the Radius on the Sines, to the Sine of (37° 24′) the Complement of (52° – 36′) the Hypothenuse.

V. Given (A E 50°) the Hypothenuse, and (A D 40° – 2′) one Leg: To find E D the other Leg.



First, *By Protraction.*

Make A D (by the IV. of the last Chap.) 40° – 2′; and thro' D (by the VI. of the last Chap.) draw (D E G) at right Angles to A D. Then at the Distance of the Length A E (by the V. of the last Chap.) describe a lesser Circle meeting D G in E: Lastly, draw A E, and the Triangle is protracted. And now the

the Leg D E, and the Angles A and E, may be meaſured by the IV. and VII. of the laſt Chapter. In this and the following, I have protracted the Triangles at the Center of the Primitive, in the four preceding I have protracted at the Primitive: But you may obſerve, that, by the Doctrine of the foregoing Chapter, you might have protracted all of theſe, either at the Center, or at the Primitive, or within it, or without it, or partly within, and partly without.

Secondly, *By the Artificial Lines.*

Becauſe the Coſine of (A D) a Leg, is to the Radius, as the Coſine of (A E) the Hypothenuſe, to the Coſine of (D E) the other Leg. The Extent on the Sines from (49° ∟ 58′) the Complement of 40° ∟ 2′, will, on the Sines, reach from (40°) the Complement of 50°, to the Sine of (57° ∟ 05′) whoſe Complement is 32° ∟ 55′, D E ſought.

VI. Given (AD, 40 ∟ 02; DE, 17° ∟ 47′) the Legs: To find the Angle A.

First, *By Protraction.*

Make (by the IV. of the laſt Chap.) A D 40° ∟ 02′, and (by the VI. of the laſt Chap.) draw D E G at right Angles to A D; then (by the aforeſaid IV. of the laſt Chap.) make DE 17° ∟ 47′, and draw AE: So is the Triangle protracted. Whence (by the IV. and VII. of the laſt Chapter) the Hypothenuſe

(A E)

(A E) and the Angles (A and E) may be meaſured: And ſince, as the Sine of one Leg (A D) is to the Radius, as the Tangent of (E D) the other Leg, to the Tangent of (A) its oppoſite Angle.

Secondly, *By the Artificial Lines.*

The Extent on the Sines from 40° ∟ 2′, to the Radius, will, on the Tangents, reach from 17° ∟ 47′, to 26° ∟ 31′. Theſe are all the various Datas in the 16 Caſes of right-angled Sphericks, and conſequently all the Varieties of Protraction. In the other 10 Caſes, whoſe Protractions are already laid down, I ſhall only propoſe them, and give the Anſwers, and leave the Operation for the Reader's Practice.

N. B. *The Figure referred to in the Laſt Solution, is to be referred to in every one of the following.*

A Synopſis

A Synopsis *of the Ten remaining Cases of right-angled spherick Triangles.*

VII. & VIII. Given the Hypothenuse AE $43^{\circ}\llcorner 9'$, and one Angle A $26^{\circ}\llcorner 31'$: To find the other Angle E, and A D, the Leg adjacent to this Angle.

1. As the Radius is to the Sine of the Complement of the Hypothenuse A E; so is the Tangent of the Angle A, to the Tangent of (20°) the Complement of (70°) the Angle E.

2. As the Radius is to the Sine of the Complement of the Angle A; so is the Tangent of the Hypothenuse A E, to the Tangent of the Side A D $40^{\circ}\llcorner 2'$.

IX. & X. Given one Leg E D $17^{\circ}\ 47'$; and the adjacent Angle 70°: To find the other Angle A, and the Hypothenuse AE.

1. As the Tangent of the Angle E, is to the Tangent of its adjacent Leg E D; so is the Radius to the Sine ($63^{\circ}\llcorner 29'$) the Complement of ($26^{\circ}\llcorner 31'$) the Angle A.

2. As the Sine of the Complement of the Angle E, is to the Radius; so is the Tangent of the Leg ED, to the Tangent of ($43^{\circ}\llcorner 9'$) the Hypothenuse.

XI. & XII. Given one Leg A D $40^{\circ}\llcorner 2'$, and its opposite Angle E 70°: To find the other Leg DE, and the Hypothenuse AE.

1. As

1. As the Tangent of the Angle E is to the Tangent of the opposite Leg A D; so is the Radius to the Sine of (17°└47′) the other Leg.

2. As the Sine of the Angle E, is to the Sine of the Leg A D; so is the Radius to the Sine of 43°└9′ the Hypothenuse AE.

XIII. Given the Angles A 26°└31′, E 70°: To find a Leg A D.

As the Radius is to the Sine of the Complement of the Angle E opposite to the Leg sought; so is the Tangent of the other Angle A, to the Tang. of (40°└2′) the Leg AD.

XIV. & XV. Given the Hypothenuse AE 43°└9′, and one Leg AD 40°└2′: To find the Angles A and E.

1. As the Tangent of the Hypothenuse A E, is to the Tangent of the Leg A D; so is the Radius to the Sine of (63°└29′) the Complement of (26└31) the Angle A adjacent to the given Leg.

2. As the Sine of the Hypothenuse is to the Radius; so is the Sine of the Leg A D, to the Sine of (70°) the Angle E opposite to this Leg.

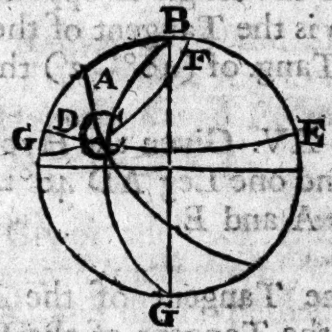
XVI. Given the Legs A D 40°└2′, D E 17°└47′: To find the Hypothenuse AE.

As

As the Radius is to the Sine of (49° ∟ 58′) the Complement of (40° ∟ 2′) A D one Leg; ſo is the Sine of (72° ∟ 15′) the Complement of (17° ∟ 47′) D E the other Leg, to the Sine of (46° ∟ 15′) the Complement of (43° ∟ 9′) the Hypothenuſe A E.

Of oblique angled ſpherical Triangles.

XVII. Given the three Sides of a ſpherical Triangle (*viz.* AB 40°, AC 46°, BC 76°); To find (A, or) either of the Angles.



First, *By Protraction.*

By the Chords make A B 40°; at the Diſtance BC (76°, from B deſcribe a leſſer Circle DCE (by the V. of the laſt Chapter); and by the ſame, at the Diſtance of AC 46°, from A deſcribe a leſſer Circle FCG, cutting the former in C. Laſtly, thro' B and C, and alſo thro' A and C, draw the great Circles BC, AC (by the II. of the laſt Chap.) and the Triangle is protracted; and then either

either of the Angles may be measured by the VII. of the last Chapter.

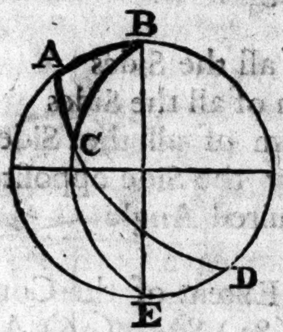
As to the Solution by the Sector, an Example considered in general Terms, will, in this Case, perhaps, be more instructive than a dry Rule, tho' that Rule be afterwards exemplified.

The Sides containing the required Angle are	A B	40	
	A C	46	
The Side opposite to the required Angle - -	B C	76	
The Sum of all the Sides		162	
Half the Sum of all the Sides		81	call it S
Half the Sum of all the Sides lessened by the Side opposite to the required Angle		05	call it D

Now the Extent of the Compasses on the Sines, from (81) S, to (46) A C, one of the containing Sides, will reach from (40) A B, the other containing Side, to another Sine, where hold the Point fast, and carry the other to D. This second Opening of the Compasses will reach on the versed Sines from the Beginning to (128° ∟ 52′) the Angle A sought. *And thus this most difficult Problem in Numbers, is readily and easily solved by the artificial right Sines, and versed Sines. Had there been nothing else to recommend these two Lines, but the Solution of this Problem alone, it were sufficient for them to claim a Place on the Sector; not only because they so neatly and expeditiously solve the Quaery, but also on account*

of the great Use of this Problem in Astronomy; and the daily Demand for its Solution by the Navigators, in order to find the Variation of the Compass; without the Knowledge of which, it would be impossible for them to steer their Course aright.

XVIII. Given two Sides (AB 40°, CB 76°) and their contained Angle (B 35° 16'): To find the third Side AC.



First, *By Protraction.*

Make (by the VIII. of the last Chapter) the Angle B 35° 16'; and on the Primitive lay AB 40°, and on CB lay 76° (by the IV. of the last Chap.) Lastly, Thro' AC (by the II. of the last) strike the great Circle AC, and the Triangle is protracted. And therefore the third Side AC, and the Angles A and C, may be measured by the IV. and VII. of the last Chap.

Secondly,

Secondly, *By the Artificial Lines.*

The Extent from the Radius, to the Sine of (54°∟44′) the Complement of (35°∟16′) the given Angle B, will reach from the Tangent of (40°) A B, one of the given Sides, to the Tangent of a fourth Arch. This fourth ſhall be leſs than 90°, if the given Angle be leſs; but greater, when greater, provided that the preceding given Side be leſs than 90°. And then the Extent from the Sine of (55°∟35′) the Complement of (34°∟25′) the preceding fourth, to the Sine of (50°) the Complement of (40°) the preceding Side, will reach from the Sine of (48°∟25′) the Complement of (41°∟35′) the Difference between the ſecond Side (76°) and the fourth Arch (34°∟25′) before found, to the Sine of (44°) the Complement of (46°) A C, the Side ſought.

XIX. Given as above: To find the two Angles A and C.

The Protraction is as before, becauſe the Data are the ſame.

		°
The given Sides containing the given Angle are - - - -	B C	76
	A B	40
Their Sum is - - - - -		116
Their Difference is - -		36
Half their Sum is - -		58
Half their Difference is - -		18

 The

The Extent on the Sines from (58°) the half Sum, to (18°) the half Difference, will reach on the Tangents from (72°∟22′) the Complement of (17°∟38′) the Half of (35° 16′) the contained Angle B, to (48°∟54′) half the Difference of the opposite sought Angles A and C. And,

The Extent on the Sines from (32°) the Complement of (58°) half the Sum of the given Sides, to (72°) the Complement of (18°) half their Difference, will reach on the Tangents from (72°∟22′) the Complement of (17°∟38′) the Half of (35°∟16′) the given contained Angle B, to (79°∟58′) half the Sum of the opposite sought Angles.

The half Sum of the opposite Angles A and C - -	79°	58′
The half Difference of A and C -	48	54
The greater Angle A - -	128	52
The lesser C - - -	31	04

But note, that as half the Sum of the Sides A B, C B, is greater or less than 90°; half the Sum of the opposite Angles A and C, shall accordingly be greater or less than 90°. *Et vice versa.*

XX. But if the same Things are given, and all the three unknown Parts (*viz.* the Base A C, and the two Angles A, C) are sought; which is very frequent in Astronomy. First (by the XIX.) find the two Angles A and C; and then, the Extent on the

Sines

Sines from (48° ∟ 54′) the half Difference of those Angles, to (79° ∟ 58′) their half Sum, will reach from the Tangent of (18°) half the Difference of the Legs BC, AC, to the Tangent of (23°) half the Base.

XXI. Given two Angles (A 128° ∟ 52′, B 35° ∟ 16′) and the interjacent Side (AB 40°): To find (AC, BC) the other two Sides. (*See the last Figure*).

First, *By Protraction.*

Make by the Chords AB 40°; and (by the VIII. of the last Chap.) describe BCE, to make the Angle B 35° ∟ 16′, and AD to make the Angle BAD 128° ∟ 52′, or the Angle DAE 51° ∟ 8′, and the Triangle ABC is protracted; and consequently the Legs AC, BC, and the Angle C, may be measured by the IV. and VII. of the last Chapter.

The Computation by the artificial Lines in this and the following, is in all respects the same with Article XIX. and XX. by changing the Angles into Sides, and the Sides into Angles, save that instead of the interjacent Side, and its opposite Angle, you must take their Supplements to 180°. For,

The given Angles - -	A	128°	52′
	C	35	16
Their Sum - - -		164	08
Their Difference - - -		93	36
Half their Sum - -		82	04
Half their Difference - -		46	48

Then the Extent on the Sines from (82° 4′) half the Sum of the given Angles, to (46° 48′) half their Difference, will reach on the Tangents from (20°) the Half of (40°) the given Side, to (15°) half the Difference of the sought Legs A C B. And,

The Extent on the Sines from (7° 56′) the Complement of (82° 4′) half the Sum of the given Angles, to (43° 12′) the Complement of (46° 48′) half the Difference of the given Angles, will reach on the Tangents from (20°) the Half of (40°) the given Side, to (61°) half the Sum of the sought Legs.

Half the Sum of the sought Legs -	61°
Half their Difference - - -	15
The greater Leg B C - -	76
The lesser A C - - -	46

N. B. *Observe the Note in the XIXth.*

XXII. Given

XXII. Given as before: To find all the three unknown Parts, *viz.* the Angle C, and the Legs AC, BC.

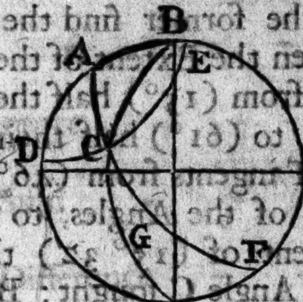
First, By the former find the Legs AC, BC: And then the Extent of the Compasses on the Sines from (15°) half the Difference of those Legs, to (61°) half their Sum, will reach on the Tangents from (46°∟48′) half the Difference of the Angles, to (74°∟28′) the Complement of (15° 32′) the Half of (31°∟4′) the Angle C sought: But the Angle C might have been found without the Legs (as in the XIX) if you had changed the Side AB, and its opposite Angle C, for their Supplements, to 180°; and then called the Angles Sides, and the Sides Angles.

XXIII. Given two Sides AB 40, AC 46, and an Angle B 35°∟16′, opposite to AB, one of them: To find the other three unknown Parts, *viz.* the Side BC, and the Angles A, C. (*See the following Figure*).

First, *By Protraction.*

Make by the Chords AB 40°; draw BG so that the Angle B may be 35°∟16′, by the VIII. of the last. At the Distance of AC 46°, describe about the Pole A, the lesser Circle DCE meeting BG in C. Lastly, thro' A and C draw the great Circle ACF, and the Triangle is laid down; and then the

unknown Parts may be measured by the IV. and VII. of the last. Now,



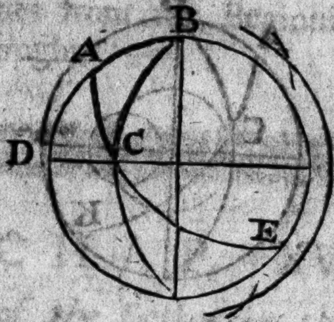
Because Sines of Angles are as the Sines of their opposite Sides, the Extent on the Sines from (46°) the Side AC, to (35° 16′) its opposite Angle B, will reach from (40°) the Side AB, to (31° 4′) the Angle C.

The other Side BC, and its opposite Angle A, may be found by the latter Parts of the XX. and XXII.

XXIV. Given two Angles, A 128° 52′, B 35° 16′; and BC 76°, a Side opposite to one of them: To find the other three unknown Parts.

Make the Angle B 35° 16′ (by the VIII. of the last Chap.) and cut off BC 76° (by the IV. of the last Chap.). Then (by the IX. of the last Chap.) from C so draw the great Circle ACE, that it may make the Angle CAB 128° 52′, or the Angle DAE 51° 8′; and so will the Triangle be constructed;

structed ; and what's unknown may be measured (by the IV. and VII. of the last Chap.)



By the Artificial Sines.

The Extent from (51° 8′) the Supplement of (128° 52′) the Angle A, to 180°, to (76°) the Side B C opposite to it, will reach from 35° 16′) the Angle B, to (46°) its opposite Side B C.

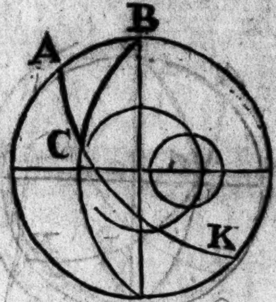
The Angle C, and its opposite Side A B, may be found by the latter Parts of the XX. and XXII.

XXV. Given all the Angles, A 128° 52′, B 35° 16′, C 31° 4′: To find a Side AC. (*See the following Figure*).

First, *By Protraction.*

Make (by the VIII. of the last Chap.) the Angle B 35° 16′; and (by the X. of the last Chap.) draw A K so that it may make the Angle A 128° 52′, and the Angle C 31° 4′; then the Triangle will be laid down, and

and the Sides may be measured by the IV. of the last Chapter.



Secondly, *By the Artificial Lines.*

The Solution is the same with the XVII. provided that you change one of the Angles adjacent to the Side sought, no matter which, into its Supplement to 180°; and then call the Angles Sides, and the Sides Angles. Here I change the Angle A.

The Angles adjacent to the sought Side are - -	51°	08′	A
	31	04	C
The third Angle -	35	16	B
The Sum of all the Angles	117	28	
Half the Sum of all the Ang.	58	44	S
Half the Sum lessened by the Angle opposite to the required Side is -	23	28	D

Now the Extent of the Compasses on the Sines from (58°L 44′) S, to (51°L 8′) A, one of the adjacent Angles, will reach from

(31°

(91° 44') C, the other adjacent Angle, to another Sine, where hold the Point fast, and carry the other Point to D. This second Opening of the Compasses will reach on the versed Sines from the Beginning, to (46°) the Side sought.

CHAP. XVII.

The Use of the Sector *in the* Projection *of the* Sphere.

First, *In the Orthographick.*

[*See Fig. of Orthographick Projection.*]

WITH any convenient Radius describe a Circle representing the general Meridian, which divide into four equal Parts by the Diameters E Q, P S. E Q representing the Equinoctial Circle, P S representing the 6 o' Clock Hour Circle, and the Axis of the Sphere. Divide each of these 4 Quarters of the Meridian into 90°, and number every 10th from E towards the Poles, and from the Poles towards Q, by the Figures 10, 20, 30, *&c.* From E and Q towards the Poles, count 23° $\frac{1}{2}$, and draw the strait Lines marked ♋ F, ♑ G; these will represent the Tropicks of *Cancer* and *Capricorn*: And parallel to these, draw strait Lines thro' every whole Degree between them; and these will represent the Parallels

of

of the Sun's Declination. The same may be done for all the other Parallels, and they will divide the Lines C P, C S, into Lines of Sines; draw ♋ C ♑, and it will represent the Ecliptick. Make C E, or C Q, Radius on the Sines; and by it, from the Sector, make C E, C Q, C ♋, C ♑, *Lines of Sines* divided into every Degree, if there be Room enough. Then, making the Half of each of these Parallels of Declination a Radius, make them *Lines of Sines* from S P, both upwards and downwards: And thro' these Divisions, at every 15th Degree of the Equinoctial, draw smoothly a crooked Line, and they will represent the Hour Lines; which, towards P, the North Pole, are numbered to answer the Morning Hours; those towards S, the Evening ones. The Semicircle S E P denoting the Mid-day Hour Circle, or 12 at Noon. Produce the Tropicks to H and K, to L and I; so shall the Spaces between K and L, H and I, be for the Kalender; K and H denoting the 10th of *December*, and L and I the 10th of *June*.

Seek in a Table what is the Sun's Declination at the End of *December*; and against that Declination draw a Line, or Division, in the Kalender; and also seek the Sun's Declination for the last Moment of *January*, and against the Parallel of this Declination, draw a Division in the Kalender, and the Month of *January* falls wholly between these two Divisions: And the like may be done for every other Month; and also for every Day, or every fifth Day, according to the Room

Room in the Instrument. Thus far the Instrument is universal.

If you would restrain it to a particular Latitude, count the Latitude in the Meridian S E P from E, according as it is North or South: In this Example it is for *London*, in the Latitude of 51°∟32′ North, represented by Z. Draw the prime Vertical Z C N, and at right Angles to it, the Horizon H C *h*, and divide them as the Equinoctial is divided; and parallel to the Horizon, at 18° below it, draw T W, it will be the Line between the enlightened and dark Hemisphere; divide it as the 18th Parallel of Declination is divided. This is used in shewing the Time of Day-break. Lastly, If the Lines Z C, H C *h*, T W, were Rulers of Brass fix'd to one another, and another is made to slide on CZ, the whole would be universal, and capable to solve all spherical Problems relating to the diurnal Motion of the Sun, and of as many of the Stars also, as you please to put on it.

Some of the more obvious Uses of this Projection.

By the Day of the Month, the Sun's Place, or right Ascension, may be found the Declination; by the right Ascension may be found the Place, *Et vice versa*. When the Latitude is (51°∟32′) known as well as the Day of the Month, which let be the 29th of *July*, when the Sun is in 16° of *Leo*, and so has 15° Declination, and his right Ascention is 8^h 54′. Then the Meridian Altitude is

H A

HA $53^{\circ}-30'$, the Midnight Depression is B*b* $23^{\circ}-30'$, the ascensional Difference is CG 1^{h} $26'$, the semidiurnal Arch is AD 7^{h} $26'$ (and so the Length of the Day 14^{h} $52'$), the seminocturnal Arch is DB 4^{h} $34'$, (and so the Length of the Night is 9^{h} $8'$), the Amplitude CD is $27^{\circ}-50'$; and from the End of Twilight, to Midnight, is 1^{h} $56'$, and so the whole Length of real Night is 3^{h} $52'$; and from Day-break, to the End of Twilight, is 20^{h} $8'$.

If the Sun's Altitude were given 17°, draw a Line thro' 17° of C z, parallel to the Horizon H *b*; and where it cuts the Parallel of the Declination, as in O, you will find by Inspection, the Hour of the Day; and the Distance R O laid on the Parallel of the Latitude of 17°, shews the Azimuth. And if the Instrument had been fitted as before directed, these latter, with others, had been done easier and exacter. This, with the Projection used in the Construction of solar Eclipses, are the Principal in the Orthographick.

Of the Stereographick Projection.

[*See Fig. of the Stereographick Projection.*]

The two following are the most remarkable; the one on the Plain of the general Meridian, and the other on the Plain of the Horizon; tho' the Sphere may be projected on the Plain of any Circle.

Describe a Circle to represent the general Meridian, quarter it, divide it, and number it as the foregoing. Lay the Tangent of half

half 75° from C towards E, which gives a Point, thro' which, and the Poles S, P, a Circle is to paſs to repreſent the Hour Circle of XI. and I.; lay the Tangent of (15°) its Complement, from C towards Q, it gives the Center; and therefore you may deſcribe the Circle. Again, Lay the Tangent of half 60°, from C towards E, which gives a Point, thro' which, and the Poles, the Hour Circle of X. and II. paſſes: And its Center is had by laying the Tangent of 30°, the Complement of 60°, downwards from C towards Q; and therefore the Circle may be deſcribed. And in like Manner you may proceed to lay the Tangents of half 45, 30, 15 upwards; and the Tangents of (45, 60, 75) their Complements downwards, to find the Centers, and then deſcribe the Circles: And, as you have drawn the Circles in one Semicircle, turn the Paper and draw them in the other Semicircle. If the Plain be large, and you have Room enough, you may bring in every 5th Degree, and you will have the Circles to every 20th Minute in Time; and every two Diviſions will anſwer to 10° in Motion. Laſtly, Subdivide by the Tangents of the half Degrees, 'till you have divided the Equinoctial E C Q into 90°. The Parallels of Latitude have their Centers in the Six o' Clock Hour Circle produced. So, would I deſcribe the Parallel of 40°, I take the Tangent of 50°, its Diſt. from the Pole; and one Foot placed on 40 in the Meridian, I turn the other about 'till it falls on the 6 o' Clock Hour Circle produced, that is its Center, therefore de-

ſcribe

scribe it. In like Manner describe the rest of them with the two Tropicks and polar Circles. The Tropick of *Cancer* is mark'd ♋ I; that of *Capricorn* ♑ H. Lastly, draw H C ♋ the Ecliptick, and divide it from C to ♋, and from C to H by the Tangents of half the Degrees: And do the like from C to P, and from C to S.

If you would restrain it to a particular Latitude, draw the prime Vertical and the Horizon, and divide them as C P is divided; 18° beneath the Horizon, draw a Parallel to determine the Beginning and End of Twilight; which, if you please, you may divide into 18 equal Parts, representing every 10th Degree of Azimuth. As to the Uses of this Projection, they are so much like the former, that I shall refer you to them: But take notice, that this, as well as the preceding, might be made universal.

The other Projection I shall shew how to lay down by the Sector, is very frequently used in Dials, both fix'd and portable. It is a Projection on the Plain of the Horizon.

[*See Projection on the Plain of the Horizon.*]

Describe a Circle, quarter it, and divide it, and number it from A to B and D, and from *a* to B and D. A *a* is the Meridian, and B C and D C the prime Verticals. Lay the Tangent of half (51° ∟ 30′) the Latitude of the Place, from C to E; then is E the Point where the Meridian and Equinoctial intersect each other. Lay the Tangent of half (38° ∟ 28′) the Complement of the Lat. from

from C towards *a*, it will give the Center to describe the Equinoctial, and it will pass thro' E, B, and D. Lay the Tangent of half (38° L 28′) the Complement of the Latitude, from C towards *a*, *viz.* to P, and it is the Pole; and therefore a Point thro' which all the Hour Circles are to pass. Lay the Tangent of the Latitude from C, on CA, and it will give CF, the Center of the 6 o' Clock Hour Circle, which will pass thro' B, P, and D. At right Angles to FC, at F draw GFH, in which are the Centers of all the Hour Circles. Make PF the Radius, and from F, both Ways, lay off the Tangents of 15, 30, 45. Then make PF a Radius on the upper Tangents, and lay off, both Ways, from F, the Tangents of 60° 75′; and these shall be the Centers to strike the Hour Circles thro' the Pole P. If these Tangents had been laid down to every 7° 30′, the half Hour Circles would have been described: And, if they had been laid down to every 10°, they would have represented Meridians to every 10°. Now make again the Radius of the Primitive, a Radius on the Chords.

The Parallels of Declination may be drawn (by the 5th Prop. of the 15th Chap.) But for this Work the Parallels are very easily described: For, take their Distance from the elevated Pole, and take also the Complement of the Latitude, and find their Sum, and also their Diff. Then lay the Tangent of half their Diff. from C towards A, it will give the Point where this Parallel cuts the Meridian; also lay the Tangent of the half

N Sum

Sum from C on Ca; and from this Point to the former, bisect the Distance, and it shall be the Center of the Parallel, which therefore may be described. The Examples shall be the two Tropicks. First, The Tropick of *Cancer* is 66° 30′ distant from the Pole, and the Complement of the Latitude 38° 30′; their Sum is 105°, and their Difference is 28°. Therefore lay the Tangent of half 28° from C to ♋, and the Tangent of half 105°, from C to K. Then find the Middle of ♋ K, which is L; from L, with the Distance L ♋, describe M ♋ N, and it is the Parallel required. Also the Tropick of *Capricorn* is distant from the elevated Pole 113° 30′, and the Complement of the Latitude is 31° 30′: Their Sum is 152°, and their Difference is 75°. Therefore lay the Tangent of half 152° from C, beyond a, to R, and the Tangent of half 75° from C to ♑. Then find the Middle, which is l, and with the Distance l ♑, from l describe S ♑ T, and it is the Parallel required. In like manner the other Parallels of Declination were described; But you may observe, that, if this Projection had been made for any Place in the *Torrid Zone*, suppose for the Latitude of 23° North, the half Tangent of C ♋, as well as of C K, must be laid on C a: And the same must be observed of every Parallel that falls between the Zenith and the elevated Pole; but where the Distance of the Parallel is 180° from the Zenith, that Parallel will be a strait Line.

The

The two Arches repreſenting the two Halves of the Ecliptick, were thus deſcribed.

The Diſtance from C to ♋, was the Tangent of half 28°; and therefore (by the IV. of the XVth Chap.) lay the Tangent of 62°, its Complement, from C to V, and it gives the Center from whence B ♋ D may be deſcribed. The Diſtance from C to ♑ was the Tangent of half 75°; therefore lay the Tangent of 15°, its Complement, from C to W, and it is the Center to deſcribe B ♑ D. Theſe two repreſent the two Halves of the Ecliptick.

For the Poles of the Ecliptick, ſince ♋ is diſtant from C by the Tangent of half 28°, lay the Tangent of half 62°, its Complement from C to X; then a Ruler laid from X to the ſeveral Diviſions in the primitive Semicircle B A D, will divide B ♋ D. And in like manner, ſince ♑ is diſtant from C by the Tangent of half 75°, lay the Tangent of half 15°, its Complement from C to *y*; and then a Ruler laid from *y* to the ſeveral Diviſions in B A D, will divide B ♑ D into the like Diviſions. Put on a Kalender between M and S, N and T, as taught in the firſt Example of this Chapter. Laſtly, divide C *a* into a Line of half Tangents running to 90°, and alſo a Line of Shadows; or rather fit a divided Scale to turn about on C.

CHAP. XVIII.

The Construction and Use of the Lines of Latitudes and Hours.

THESE Lines are placed near the Edges of the Sector, and properly belong to the plain Scale, but are often laid down on the Sector for their Use in readily drawing the Hour Lines on any plain Dial.

The *Line of Hours* is thus constructed:

Having resolved on your Radius, make it a Parallel at the Tangent of 45. Take the parallel Tangent of 15°, and lay it both Ways from III. to II. and IV. And lay the parallel Tangent of 30° from III. to I. and V. And of 45° from III. to o and VI. And so the whole Hours will be laid down.

If you would draw it to every 5th Minute, or nearer, you may do it by allowing 15° to every Hour, or 15′ a Quarter of a Degree, in Motion to every Minute in Time. And it is evident, that this Line is two Lines of Tangents, running upwards and downwards from the Middle, numbered III. but reduc'd to Time.

The

The *Line of Latitudes* may be constructed thus:

Make the whole Line of Hours the Radius of a Semicircle ABDC. At the Ends of the Diameter raise the Perpendiculars AE, DF, and make them *Lines of Sines* to the same Radius.

From the Center, to the several Divisions in these Sines, draw strait Lines, meeting the Periphery, in as many Points, which number with the Numbers correspondent to them on the *Lines of Sines*.

Raise CB perpendicular, and lay a Ruler cross it from the like Numbers in the Arch, and it will cut CB into a Line of Latitudes. *See Fig.* 63.

The *Line of Latitudes* may be thus constructed:

Take the *Line of Hours* and make it a parallel Sine of 90°. Then, for any Division, suppose that of 35°. Take the crural Sine of 35°, and lay it on the crural Tangent, and you will find it give on the Tangents 29°∟50′; the parallel Sine of which gives the Distance of 35° from the Beginning of the Line: And so for any other Degree.

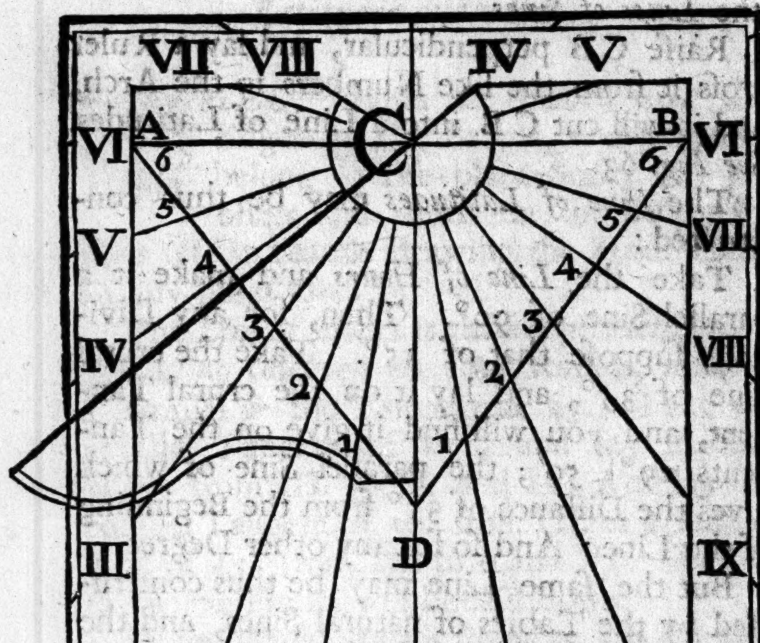
But the same Line may be thus constructed by the Tables of natural Sines, and the *Line of Lines* on the Sector.

The natural Sine of 35° is 5735764, which sought among the Tangents, gives 29°∟50′, and the natural Sine of 29°∟50′ is 4974787. Open the Sector 'till the *Line of Hours* is a Parallel to the whole *Line of Lines*. And then the Parallel of 4974787, will be the

Diſtance to be laid down from the Beginnin g of the Line, to the Diviſion repreſenting 35°: And in the ſame Manner may any other Diviſion be laid down.

The Uſe of theſe Lines will be manifeſt from the two following Examples.

1*ſt*. To draw an horizontal Dial for any Latitude, ſuppoſe 5, $\frac{1}{2}$ N. Draw C, XII, to repreſent the Meridian, and it is the Hour



Line of 12. Croſs it at right Angles by AB, and it gives the Hour Lines of 6 and 6. Take from the *Line of Latitudes*, the Latitude of the Place, ſuppoſe that of *London* 51 $\frac{1}{2}$ N; and lay

lay it on the 6 o' Clock Hour Lines from C to A and B. Take the whole *Line of Hours* in your Compaſſes, and one Foot in B, or A, turn the other about 'till it falls in the Meridian, as here in D; and draw D A, D B; and on D A, D B, transfer the Diviſions from the *Line of Hours*. Then Lines drawn from C thro' thoſe Diviſions, will be the Hour Lines ſought. The Stile meets the Plain in C, and ſtanding over the Meridian C, XII, makes an Angle with it of 51 $\frac{1}{2}$, the Latitude of *London*. The Hours 4, 5, before 6 in the Morning, and 7 & 8 after 6 in the Evening, are drawn by continuing their oppoſite Hour Lines 4, 5 in the Afternoon, and 7, 8 in the Forenoon, beyond the Center. In like manner may a direct South or North Dial be drawn, omitting thoſe Lines that would be uſeleſs.

2dly. To deſcribe a vertical declining Dial.

Let the Example be for a South Dial, declining 20° Weſtward, in the Latitude of 51 $\frac{1}{2}$ North.

In order to this, there are three Things neceſſary, which may very readily be obtained by the artificial Sines and Tangents.

1ſt. The Extent from the Sine of (51 $\frac{1}{2}$) the Latitude to the Sine of 90°, will reach from the Tangent of (20°) the Declination, to the Tangent of the Plain's Difference of Longitude 24° 57′.

2dly. The Extent from the Sine of 90°, to the Sine of 65° ∟ 3′, the Complement of 24° ∟ 57′, the Plain's Difference of Longitude, will reach from the Tangent of 38° ½, the Complement of (51° ½) the Latitude, to the Tangent of 35° ∟ 48′ the Stile's Height.

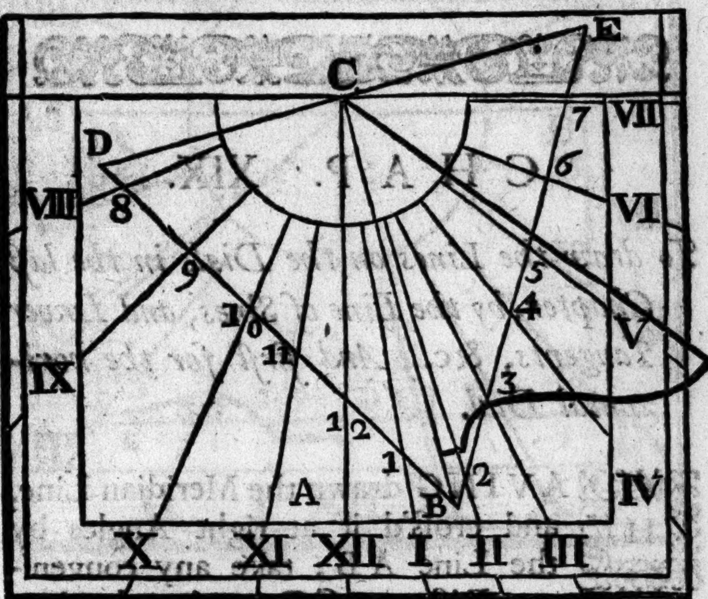
3dly. The Extent from the Radius to the Sine of the Stile's Height, will reach from the Tangent of (24° ∟ 57′) the Plain's Difference of Longitude, to the Tangent of (15° ∟ 13′) the Substile's Distance from the Meridian.

Now, because the Plain's Difference of Longitude is 24° ∟ 57′, which reduced to Time, is 1^{h} 40′ nearest, I perceive that the Substile falls between the one and two o' Clock Hour Lines: And one Hour being taken away, in order to find the Hour Line, next to the Substile you have left 40′.

Draw A C perpendicular to the Horizon, to represent the 12 o' Clock Hour Line. With it from C make an Angle of 15° ∟ 13′, by the Line of Chords, by drawing the Line C B; and this must be to the right Hand, because the Declination is Westward. This C B is the substilar Line. Draw D C E perpendicular to C B; and from the Line of Latitudes, take 35° ∟ 48′, the Stile's Height, and lay it from C to D. Take the whole Line of Hours in your Compasses, and with one Foot in D, turn the other about 'till it falls on the Substile C B, as here in B, and

draw

draw D B. Make the Angle C B E equal to the Angle C B D. Transfer the Divisions of the *Line of Hours*, on D B and B E: Then count from B towards D; and also



from E towards B, 40′, 1^h 40′, 2^h 40′, 3^h 40′, 4^h 40′, 5^h 40′, &c. and thro' these and C, shall the Hour Lines be drawn: And in like manner might the Halfs and Quarters be laid down.

For Example; The Plain's Difference of Longitude is 1^h 40′, from which take away all the Hours, Halfs, and Quarters, and there will be left 10′; to this add continually 15′, and you have 10′, 25′, 40′, 55′, 1^h 10′, 1^h 25′, 1^h 40′, &c. to prick from the *Line of Hours* on the Lines B D, E B.

And

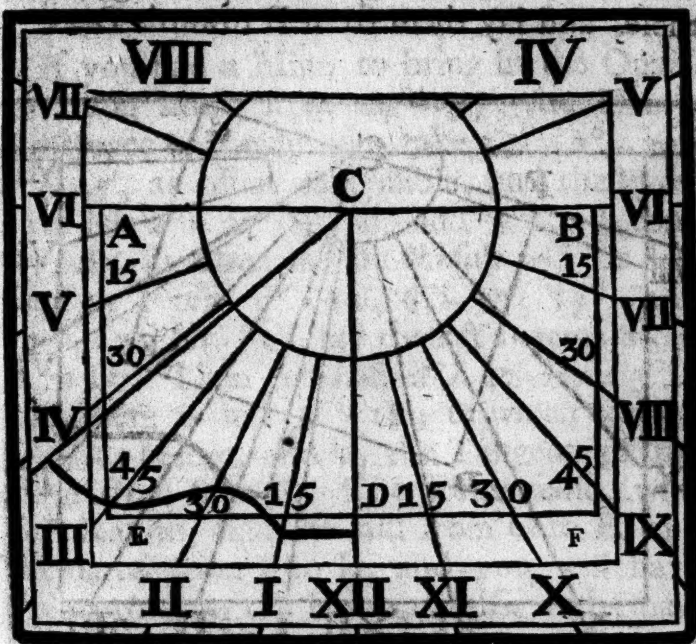
And then Lines drawn thro' these Points and C, are the Lines for the Hours, Halfs, and Quarters. Lastly, the Stile must meet the Plain at C, and standing on C B, must make an Angle with it of 35° 48'.

CHAP. XIX.

To draw the Lines on the Dials in the last Chapter, by the Line of Sines, and Lower Tangents, &c. *And first for the horizontal Dial.*

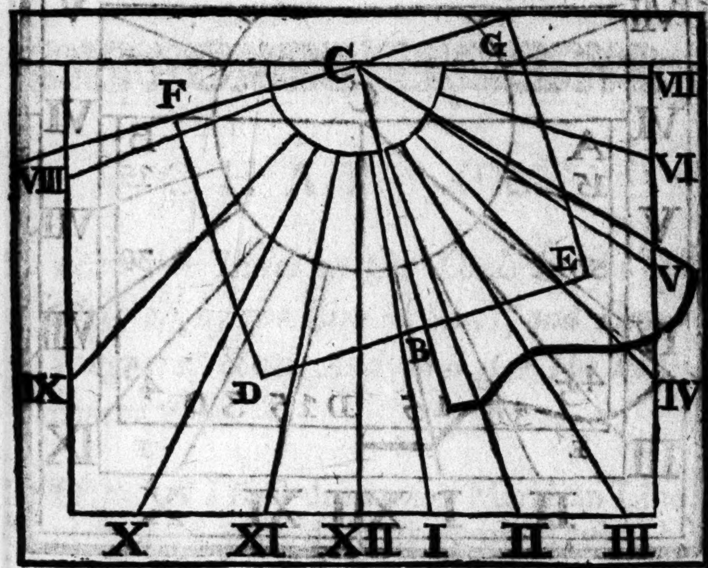
HAVING drawn the Meridian Line, and cross'd it at right Angles by the Line A B; take any convenient Distance C D, and make it a parallel Sine of 90°. Then lay the Sine of the Latitude from C to A and B: And compleat the Rectangles A C D E, B C D F; the Sector still retaining the same Opening, lay the Tangent of 15°, 30°, 45°, from A and B, to E and F. Now, making DE, or DF, Radius, lay the Tangent of 15°, 30°, 45°, from D on D E and D F. Lastly, from C, thro' these Points, draw Lines, and they will be the Hour Lines. In like manner the Halfs and Quarters, or Minutes, may be laid down, allowing according to the Proportion

portion of 15° to an Hour, 7°∟30′ for half an Hour, and ſo on.



2*dly*. For the declining Dial. Having, as in the laſt Chapter, prick'd down the ſubſtilar Line, take any convenient Diſtance from C thereon, as CB for the Radius; and croſs it at right Angles in B and C. Then make C B a parallel Radius on the Sines, and take the parallel Sine of 35°∟48′, the Stile's Height, and lay it from B to D and E, and from C to F and G, and draw F D, G E, compleating the Rectangle D E G F. Make DB the Radius of a Line of Tangents. The Plain's Difference of Longitude being 24°∟57′, take from it 15° as often as poſſible, and there will be left 9°∟57′; to this add 15° as often

often as the Reſult will be leſs than 45°, and you will have 9°∟57′, 24°∟57′, 39°∟57′. To the Radius B D, take the Tangents of theſe, and lay them from B towards D.



3*dly*. Take 9°∟57′ from 15°, and there is left 5°∟3′; to this add 15° continually, as often as the Reſult will be leſs than 45°, and you will have 5°∟3′, 20°∟3′, 35°∟3′: Of theſe, to the former Radius B D, take the Tangents and lay them from B towards E.

4*thly*. Make F D, or G E, Radius, and lay the Tangents of the former Degrees, *viz.* of 9°∟57′, 24°∟57′, 39°∟57′, from G towards E: And the Tangents of the latter Degrees, *viz.* of 5°∟3′, 20°∟3′, 35° ∟3′, from F towards D.

Laſtly,

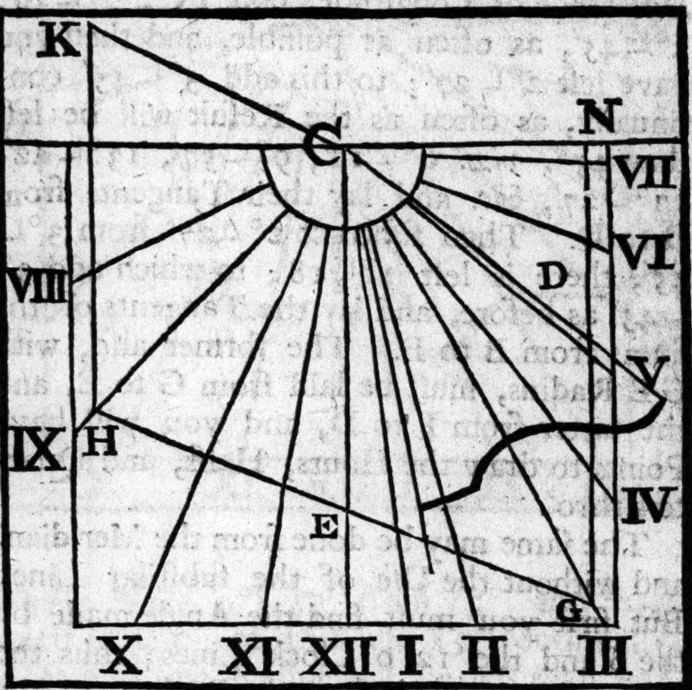
Lastly, Thro' these Points from C, strait Lines drawn are the Hour Lines.

If you had a Mind to bring in the Quarters, you must, from $24^{\circ}\llcorner 57'$, the Plain's Difference of Longitude, take 15°, $7^{\circ}\llcorner 30'$, $3^{\circ}\llcorner 45'$, as often as possible, and then you have left $2^{\circ}\llcorner 27'$; to this add $3^{\circ}\llcorner 45'$ continually, as often as the Result will be less than 45°, *viz.* $6^{\circ}\llcorner 12'$, $9^{\circ}\llcorner 57'$, $13^{\circ}\llcorner 42'$, $17^{\circ}\llcorner 27'$, *&c.* and lay their Tangents from B to D. Then subtract $2^{\circ}\llcorner 27'$ from $3^{\circ}\llcorner 45'$, there is left $1^{\circ}\llcorner 18$; to which add $3^{\circ}\llcorner 45'$ as before, and lay the Tangents of the Sums from B to E. The former also, with GE Radius, must be laid from G to E, and the latter from F to D, and you will have Points to draw the Hours, Halfs, and Quarters thro'.

The same may be done from the Meridian, and without the Use of the substilar Line: But first you must find the Angle made by the 6 and the 12 o' Clock Lines; thus the Extent on the Sines from the Radius, to the Sine of the Declination, will reach from the Tangent of the Latitude, to the Tangent of ($23^{\circ}\llcorner 16'$) the Complement of ($66^{\circ}\llcorner 44'$) the Angle sought.

Having drawn C E the Hour Line of 12, make with the Chords the Angle E C D (66° $44'$) equal to that made by the 12 and 6 Hour Lines. And lay C D to the right Hand, because the Declination is South-West. With any convenient Radius, make C E the Sine of

of (70°) the Complement of (20°) the Declination: And CN the Sine of (38° 30′) the Complement of (51 ½) the Latitude. And draw N G parallel to C E, cutting C D in D.



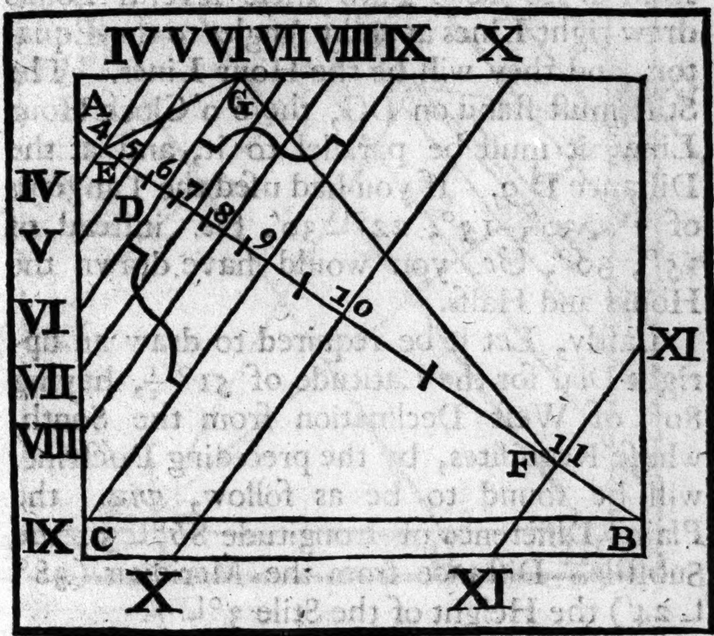
Compleat the Parallelogram C D G E; continue G E 'till E H be equal to it; and draw from H a Line parallel to C E. Make G E, or E H, a Radius on a Line of Tangents; and lay from E towards G and H, 15° and 30′, for the Hours of 10, 11, 1, 2. Alſo make D G Radius, and lay the Tangents of 15° and 30°, from D and H towards G and N, and divide H K, as G N is divided, as far as the

*

the Plain will bear it. And draw Lines from C to these several Divisions, and also to the Angles G and H, and you have the Hour Lines required. The Stile is to form an Angle, and to be seated as in the last Operation. You might have proceeded to lay down the Halfs and Quarters, or, indeed, the Minutes, as in the first Example of this Chapter.

Here hath been considered the Use of the Sector in drawing the Hour Lines on Dials, having the Pole considerably elevated above them: It remains to shew how, by the same Lines, to draw Dials that are polar, or have the Pole very little elevated above them.

The Example shall be for a full East Dial, in the Latitude of 51 $\frac{1}{2}$.



Draw

Draw the horizontal Line C B, and, by the Chords, make the Angle C B A (38 $\frac{1}{2}$) the Complement of the Latitude, and A B will represent the Equator. Chuse two Points, E and F, near the Extremities of A B, thro' which draw the Lines XI, XI; IV, IV, at right Angles to A B: And they may represent the Hours of 4 and 11 in the Morning. Make, by the Help of the Chords, the Angle EFG 15°, and the Angle FEG 60°. Make G E a Radius on the Sines, and lay the Sine of 30° from E to D, and draw D G. Make G D a Radius on the Tangents, and lay the Tangent of 15° from 6 to 5 and 7; and the Tangent of 30° from 6 to 4 and 8: Of 45° from 6 to 9. Make G D the Tangent of 30°, and lay the Radius corresponding to it, from D to 10. Thro' these several Points draw right Lines at right Angles to the Equator, and they will be the Hour Lines. The Stile must stand on DG, the 6 o'Clock Hour Line; it must be parallel to it, and at the Distance D 9. If you had used the Tangents of 7° ∟ 30', 15°, 22° ∟ 30', &c. instead of 15°, 30°, &c. you would have drawn the Hours and Halfs.

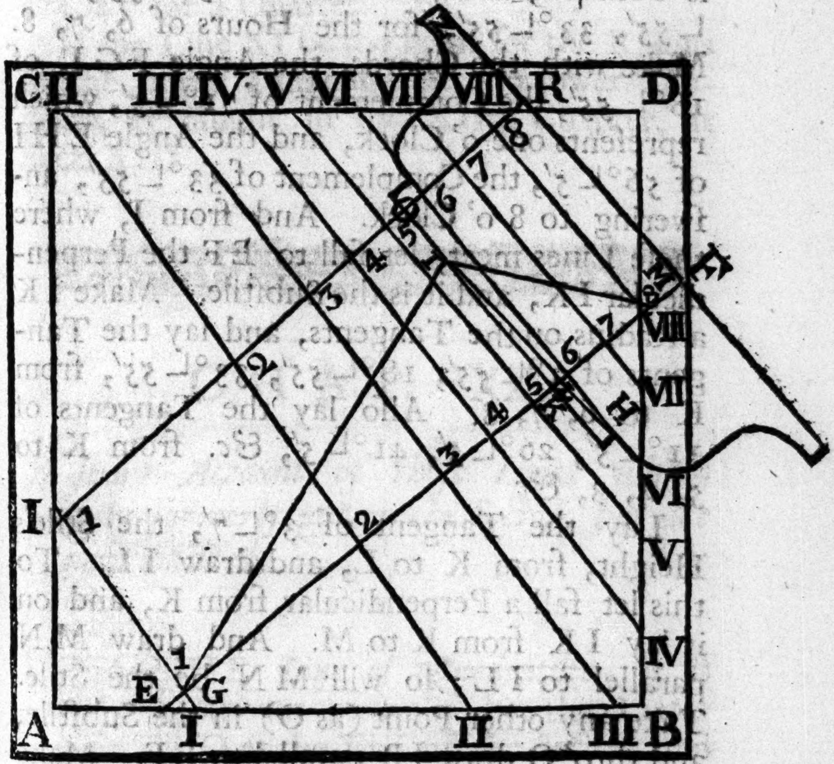
Lastly, Let it be required to draw an upright Dial for the Latitude of 51° $\frac{1}{2}$, having 80° of West Declination from the South, whose Requisites, by the preceding Doctrine, will be found to be as follow, *viz.* the Plain's Difference of Longitude 86° ∟ 5', the Substile's Distance from the Meridian (38° ∟ 24') the Height of the Stile 3° ∟ 7'.

If

If the Meridian Line be introduced on this Dial, almost all the rest of the Hour Lines will fall so very near the Substile, that they will be of very little Use. It is therefore the best Way to bring in some of the more useful Lines at a competent Distance, tho' you lose one or two, which would discommode the Dial in general.

In this Example, let it be proposed to bring on all the Lines, after one o' Clock.

Let the Plain on which the Dial is to be drawn, be A B C D. In the horizontal Line



AB, take the Point E at Pleasure towards the left Hand, because it is a South-west Decliner;

 and,

and, with a *Line of Chords*, make the Angle F E B of (38° 24′) the Subſtile's Diſtance from the Meridian: So will E F be the Equator. Aſſume any two Points, G and H, in it to denote the Place of the extream Hour Lines in this Equator. From (86° 5′) the Plain's Difference of Longitude, ſubtract 15° continually, and you will have the Remainders 71° 5′, 56° 5′, 41° 5′, 26° 5′, 11° 5′, anſwering to the Hours 1, 2, 3, 4, 5. And 11° 5′ taken from 15°, leaves 3° 55′; to which add 15° as often as there is Occaſion, and you will have 3° 55′, 18° 55′, 33° 55′, for the Hours of 6, 7, 8. Make with the Chords, the Angle F G I, of 18° 55′, the Complement of 71° 5′, which repreſents one o' Clock, and the Angle E H I of 56° 5′, the Complement of 33° 55′, anſwering to 8 o' Clock. And from I, where theſe Lines meet, let fall to E F the Perpendicular I K, and it is the Subſtile. Make I K a Radius on the Tangents, and lay the Tangents of 3° 55′, 18° 55′, 33° 55′, from K to 6, 7, 8. Alſo lay the Tangents of 11° 5′, 26° 5′, 41° 5′, &c. from K to 5, 4, 3, &c.

Lay the Tangent of 3° 7′, the Stile's Height, from K to L, and draw I L. To this let fall a Perpendicular from K, and on it lay I K from K to M. And draw M N parallel to I L; ſo will M N be the Stile. Take any other Point (as O) in the Subſtile, and thro' O draw I R parallel to E F. Make the neareſt Diſtance from O to the Stile M N, a Radius on the Tangents. And then lay the

the Tangents of $3^{\circ}\llcorner 55'$, $18^{\circ}\llcorner 55'$, $33^{\circ}\llcorner 55'$, from O to 6, 7, 8; and the Tangents of $11^{\circ}\llcorner 5'$, $26^{\circ}\llcorner 5'$, $41^{\circ}\llcorner 5'$, *&c.* from O to 5, 4, 3, *&c.* Lastly draw right Lines thro' 1, 1; 2, 2; 3, 3; 4, 4, *&c.* and they will be the Hour Lines required. The Stile must stand right over IK; its nearest Distance from K, must be KM equal to IK; and its nearest Distance to O, must be OR, the Radius used in laying down the Divisions on PQ. In this, as in other Dials, the Halfs and Quarters might have been brought in by allowing $7^{\circ}\llcorner 30'$ for half an Hour, and $3^{\circ}\llcorner 45'$ for a Quarter.

If the Line KI be too great for the Radius of the upper Tangents, use the lower Tangents, as taught in the XIIth Chapter, at Sect. XIII.

CHAP. XX.

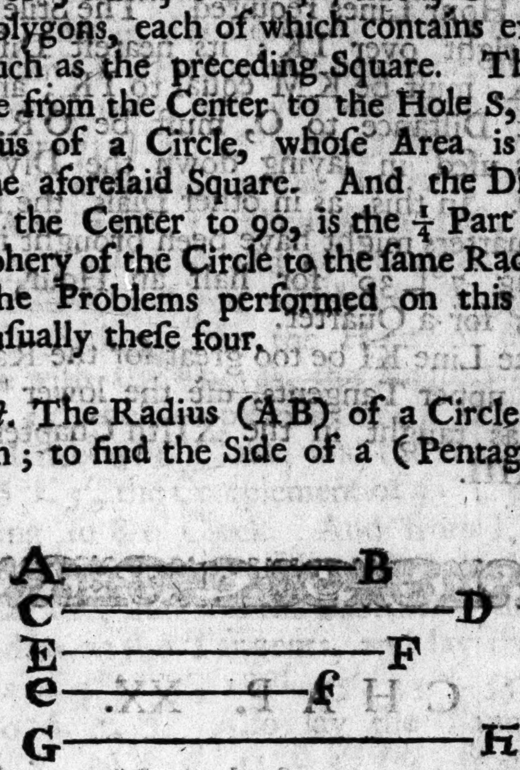
A short Account of those Lines which were formerly put on the Sector, but are now omitted.

I. THE *Line of Quadrature*; it is known generally by the Letter Q at the End of the Sector, and its punch'd Holes mark'd Q, 90, 5, 6; S, 7, 8, 9, 10, downwards.

From the Center to the Hole Q, is the Side of a Square equal to the whole Length of the *Line of Lines*. And from the Center to the Holes 5, 6, 7, 8, 9, 10, give the Sides of 5 sided, 6 sided, 7 sided, *&c.* regular Polygons, each of which contains exactly as much as the preceding Square. The Distance from the Center to the Hole S, is the Radius of a Circle, whose Area is equal to the aforesaid Square. And the Distance from the Center to 90, is the $\frac{1}{4}$ Part of the Periphery of the Circle to the same Radius.

The Problems performed on this Line, are usually these four.

1*st*. The Radius (AB) of a Circle being given; to find the Side of a (Pentagon, or



other) regular Polygon of equal Area with the Circle: Make AB a Parallel at S. And then the Parallel at the Hole numbered (5) answerable to the Number of the Sides of the Polygon, is (CD) the Side of the Polygon sought.

2*dly*. Given CD the Side of (a Pentagon, or) any regular Polygon; to find the Side of

(a Hexagon,

(a Hexagon, or) any other regular Polygon, or the Radius of a Circle, having Area's, each equal to the Area of the Polygon given. Make (CD) the Side of the given Polygon, a Parallel at (5) the Number denoting its Sides; then will the Parallel at (6 or 8) the Number denoting the Sides of the Polygon, be (EF, *ef*) the Side sought; and the Parallel at S will be (AB) the Radius sought.

3*dly*. Given (AB) the Radius of a Circle; to find the Length of a quadrantal Arch of that Circle. Make (AB) the given Radius, a Parallel at S; and then the Parallel at 90 shall be (GH) the quadrantal Arch sought.

4*thly*. Given (GH) the Length of a quadrantal Arch of a Circle; to find the Radius. Make (GH) the Length of the quadrantal Arch, a Parallel at 90; and the Parallel at S will be (AB) the Radius sought.

II. *The Line of inſcribed Bodies*. It is known by punch'd Holes mark'd T, O, C, I, S, D (downward towards the Center) denoting by their Diſtances from the Center, the Proportions of the Sides of the 5 regular Bodies inſcribed in a Sphere, whoſe Radius is equal to the Diſtance from the Center to S. So T, O, C, I, S, D, denote the Tetrahedron, the Octohedron, the Cube, the Iſcoſahedron, the Semidiameter of the Sphere, the Dodecahedron.

The Problems performed on this Line, are theſe two.

 1*ſt*. The

1*st*. The Radius (IK) being given; to find the Side of (an Octohedron, or of) any of the 5 regular Bodies inscribed in it. Make IK a Parallel at S; and then a Parallel at (O) the Letter denoting the regular Body, will be (LM) the Side of it.

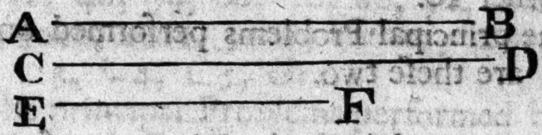
2*dly*. The Side (LM) of (an Octohedron, or) any regular Body being given, to find the Side of (an Iscosahedron) any other of the regular Bodies; or the Semidiameter of the Sphere, in which they may be both inscribed. Make LM a Parallel at (O) the Letter denoting the regular Body, whose Side is given; and then will the Parallel at (I) the Letter denoting the regular Body, whose Side is sought, be (PQ) that Side sought; and the Parallel at S will be the Radius of that Sphere, in which they may be both inscribed.

III. The *Line of equated Bodies*; It is known by punch'd Holes, mark'd T, O, S, C, I, D (downward towards the Center) denoting by their Distances from the Center, the Proportions of the Sides of the five regular Bodies, to the Distance of S from the Center,

Center, which is the Diameter of a Sphere, to which each of theſe Bodies is equal.

The Problems performed on this Line, are two.

1ſt. Given (AB) the Diameter of a Sphere; to find the Side of (an Octahedron, or of)



any one of the five regular Bodies, equal to the Sphere, whoſe Radius is given. Make (A B) the Diameter of the Sphere, a Parallel at S; and then the Parallel at (O) the Letter denoting the regular Body, will be (C D) its Side ſought: And this regular Body will be equal to the Sphere, whoſe Diameter is A B.

2*dly*. Given (C D) the Side of (an Octohedron, or) any regular Body; to find the Side of (an Iſcoſahedron, or) any other of the regular Bodies; or the Diameter of a Sphere, in ſuch Sort, that each of theſe three ſhall be equal to one another. Make C D a Parallel at (O) the Letter denoting the regular Body, whoſe Side is given; and then will the Parallel at (I) the Letter denoting the regular Body, whoſe Side is ſought, be (E F) that Side ſought; and the Parallel at S, will be the Diameter of that Sphere,

to which each of these regular Bodies is equal.

IV. The *Line of Segments*. It is known by being divided into 5 unequal Diſtances, numbered 10, 9, 8, 7, 6, 5, from the End of the Lines downwards towards the Center; each of theſe Diſtances are again ſubdivided into other 10.

The principal Problems performed on this Line, are theſe two.

1*ſt*. Given (G H) the Diameter of a Circle, and (I K) the Depth of the greater Segment; to find the Proportion that the whole

Circle bears to the greater Segment. Make (G H) the Diameter, a Parallel at 100, and carry (I K) the Depth of the greater Segment, along the Lines 'till it becomes a Parallel; which will be at 80: to which 100 bears ſuch Proportion, as the whole Circle doth to the greater Segment.

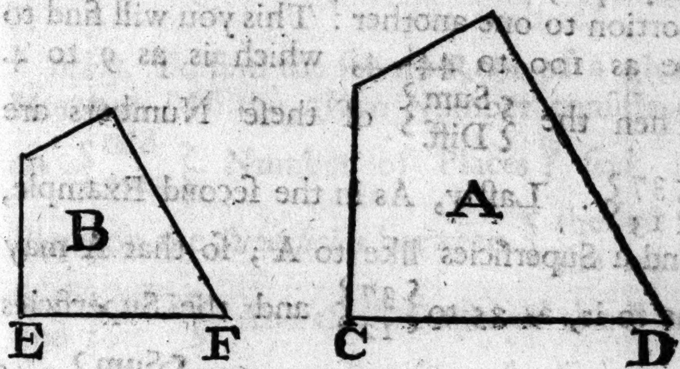
2*dly*. Given (G H) the Diameter of a Circle; to cut it ſo that the whole Circle ſhall be to the greater Segment, as 100 to 80. Make G H a Parallel at 100, and the Parallel at 80 will be I K, the Depth of the greater Segment.

V. The

V. The *Line of Superficies*; From the Center it is numbered at the grand Divisions by 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, reaching as far as the *Line of Lines* reaches; and each of these are again subdivided to suit with the decimal Order, according to the Length of the Sector, and may be read thus, 1, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100; or, 100, 1000, 2000, 3000, *&c.* or, ∟01, ∟1, ∟2, ∟3, ∟4, ∟5, *&c.*

The principal Problems performed by this Line, are these.

1st. To find the Proportion between (A, B) two similar Superficies. Make on the



Line of Superficies, any Side (CD) of the greater Superficies (A) a Parallel at 100; then carry (EF) a like Side of the other Superficies (B) along the *Line of Superficies*, 'till it becomes a Parallel, which you will find to be at 44∟4. Therefore A is to B, as 100 to 44∟4.

2dly. A

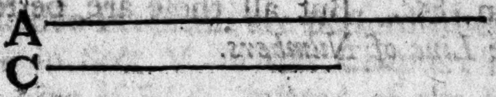
2dly. A Superficies (A) being given; to find (B) another like it; so that (A) the former, may be to (B) the latter, in a given Proportion (*viz*. as 4 to 9). Make (C D) any Side of (A) the given Superficies, a Parallel at 9, the Number answering to (A) that Superficies; and (E F) the Parallel at (80) the Number answering to the sought Superficies, will be a like Side of that sought Superficies.

3dly. Two like Superficies (A, B) being given; to find a third like to them, and equal to their $\left\{\begin{matrix}\text{Sum}\\ \text{Difference}\end{matrix}\right\}$. Seek, as in the first Example, Numbers representing their Proportion to one another: This you will find to be as 100 to 44⌞4, which is as 9 to 4. Then the $\left\{\begin{matrix}\text{Sum}\\ \text{Diff.}\end{matrix}\right\}$ of these Numbers are $\left\{\begin{matrix}37\\ 13\end{matrix}\right\}$. Lastly, As in the second Example, find a Superficies like to A; so that A may be to it, as 25 to $\left\{\begin{matrix}37\\ 12\end{matrix}\right\}$ and the Superficies thus found, will be equal to the $\left\{\begin{matrix}\text{Sum}\\ \text{Diff.}\end{matrix}\right\}$ of the Superficies.

4thly. Between two given Lines (A and C) to find a mean Proportional: Find the Numbers expressing their Proportion by measuring them on the *Line of Lines*, which you will find to be 75 and 48; then I make, on the

Super-

Superficies, C a Parallel at 75; and the Parallel at 48 is the mean Proportional sought.



5thly. To find a mean Proportional between two given Numbers. Take from the *Line of Lines*, two Lines answering to the Numbers; and, by the last, find a mean Proportional between these Lines; this mean proportional Line measured on the *Line of Lines*, gives the mean proportional Number sought.

6thly. To find the square Root of a given Number. If the given Number consists of an { odd / even } Number of Places; seek its Place on the *Superficies* between { the 1st 1 & the 2d 1 / the 2d 1 and 10 }. And the Distance of this Place from the Center, applied from the Center on the *Line of Lines*, will give the square Root sought, which will consist of half as many Places as the given Number consists of, if the Multitude of those Places be even; but if odd, of half as many Places as *the given Number increased by one*, consists of. So the square Root of 25 is 5; of 250 is 15⌊8 nearest; of 2500 is 50; of 25000 is 158. This

This is perform'd without opening the Sector. Besides these already mentioned, there are others; as three Numbers given; to find a fourth in a duplicated, or subduplicated Proportion, &c. But all these are better done by the *Line of Numbers*.

VI. The *Line of Solids*; From the Center it is numbered at the grand Divisions by 1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; and these are again subdivided to suit with the decimal Order, according to the Length of the Sector; and may be read thus, 1, 10, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000; or, 1000, 10000, 100000, 200000, 300000, 400000, &c.

The principal Problems performed by this Line, are these.

1*st*. To find the Proportion between (A,B) two similar Solids.

2*dly*. A Solid (G) being given; to find (H) another like it; so that the former may be to the latter, in Proportion of two given Numbers (of 36 to 80).

3*dly*. Two similar Solids (A, B) being given; to find a third like to them, that shall be equal either to the Sum, or the Difference of those given.

4*thly*. Between (A and C) two given Lines; to find two mean Proportionals.

5*thly*. To

5thly, To find two mean Proportionals between two given Numbers.

These five Problems are performed exactly as those in Sect. V. in this Chapter, if you use the *Line of Solids*, instead of the *Line of Superficies*. And so will the following, which is to extract the Cube Root of a given Number, provided that you seek the given Number between the $\left\{\begin{array}{l}\text{1st. 1 \& the 2d. 1}\\ \text{2d. 1 \& the 3d. 1}\\ \text{3d. 1 \& the 10}\end{array}\right\}$ when that given Number consists of $\left\{\begin{array}{l}\text{1 or 4 or 7}\\ \text{2 or 5 or 8}\\ \text{3 or 6 or 9}\end{array}\right\}$ Places; always observing, that the Root will consist of $\left\{\begin{array}{c}1\\ 2\\ 3\end{array}\right\}$ Places, when the given Number consists of $\left\{\begin{array}{l}\text{1 or 2 or 3}\\ \text{4 or 5 or 6}\\ \text{7 or 8 or 9}\end{array}\right\}$ Places. Also three Numbers being given, a fourth may be found in a triplicated, or subtriplicated Proportion. Let the three Numbers given be 4, 8, 9. Make 9 taken from the Solids, a Parallel at 4 on the *Line of Lines*, and then will the Parallel of 8 be, when measured on the Solids, 72, the Answer.

Ex. II. Let there be proposed the three Numbers 27, 125, 6; to find a fourth in a subtriplicated Proportion. Make from the *Line of Lines*, 6 a Parallel at 27 on the given

Solids; and then the Parallel at 125, will give, when meaſured on the *Line of Lines*, 10, the Anſwer.

VII. The *Line of Metals.* It hath 7 punch'd Holes mark'd ⊙ ☿ ♄ ☾ ♀ ♂ ♃, denoting Gold, Quickſilver, Lead, Silver, Copper, Iron, Tin. By their Diſtances from the Center of the Sector, is ſhewn the Proportions of their ſpherical Bodies (or of their homologous Lines) when they have equal Weights.

The Problems uſually performed by this Line, in Conjunction with the *Line of Lines*, and the *Line of Solids*, are theſe.

1*ſt*. To find their ſpecifick Gravities; that is, to find the Proportions that the Weights of theſe Metals bear to one another, in Bodies like, and equal in Magnitude. Suppoſe a Body of Gold weighs 50 Ounces, I demand the Weight of a like and equal Body of Silver. Take from the Solids (50) the given Weight of the Gold, and make it a Parallel at the Points belonging to (Silver) the other Metal; then will the Parallel at the Point belonging to (Gold) the firſt Metal, meaſured on the Solids, give (27∟2) the Weight ſought of the Silver Body. And ſo the Weights of the others will be as follow, Quickſilver 35∠7, Lead 30∠3, Copper 23∠7, Iron 21∠1, Tin 34∠5.

2*dly*. Of two like Bodies of equal Weights, but of different Metals (Gold and Silver)

(40) a Line of the (Gold) one being given; to find a like Line of (Silver) the other. Make the given Side of the Body a Parallel at the Points belonging to his Metal, and the parallel Diſtance of the other Body's Metal, meaſured on the *Line of Lines*, will give the Line ſought, 4643.

3*dly*. In like Bodies of equal Weight, (Gold and Silver) having A the Magnitude of the golden one; to find the Magnitude of the other of Silver. Take A from the *Line of Solids*, and make it a Parallel at the Point of (Gold) the Metal to which A belongs. And then the Parallel at the Point of Silver, meaſured on the Solids, will give the Magnitude of the Silver. And ſo the Parallel at the Lead, will give the Magnitude at the Lead, &c. So if Bodies were of equal Weights, and the Magnitude of the Gold be 389, that of the Quickſilver will be 549, that of Lead 643, that of Silver 716, that of Copper 822, of Iron 925, and of Tin 1000.

4*thly*. Of two like Bodies, A and B, of given different Metals, the Weights of both, C, D, being given, and a Line E of one of them; to find a like Line of the other. Firſt find, by the ſecond of this, a Line (F) for a Body (G) of equal Weight (C) with the firſt, but of the ſame Metal with the ſecond. Then are there known the Weights C and D of two Bodies, C and B of the ſame Metal, and a certain Line (F) of the former (G) is given, to find a like Line of the latter (B).

This

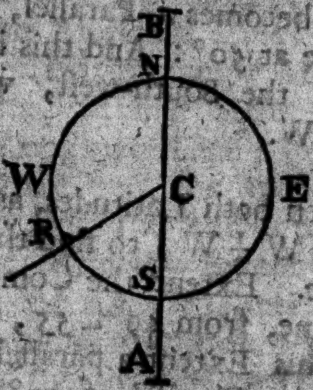
This may be done by the second Example of the VIth Section of this Chapter.

VIII. The *Tangent Line*, *Secant Line*, *Rumb Line*, and *Meridian Line*, are Lines properly belonging to the plain Scale, but are sometimes laid down on the Sector.

The Tangents and Secants are sometimes used in Projections, and sometimes in the Calculation of Triangles, in Conjunction with the *Lines of Sines* and *Lines*: But these, as hath been declared, are better done by other Methods. The *Line of Rumbs* is used in Navigation, in plotting a Traverse, and thereby exhibiting to the View, a Representation of the Ship's Way. It generally runs to 8 Points of the Compass, where it is equal to the Chord of 90° of that Circle from whence it was made; and is no other than a *Line of Chords* of the Arches of a Quadrant, divided into 8 equal Parts. There is a small Hole punch'd in it at the Beginning, and another a little beyond 5 Points and $\frac{1}{4}$. The Distance of these Holes, is equal to the Radius of the Circle answering to this *Line of Rumbs*. Its Uses are these two, *viz.* The Meridian being given, to draw a required Rumb; or, The Meridian and Rumb being both drawn, to determine what Rumb it is.

1*st*. The Meridian A B being given; to draw from it the Rumb called S W *b* W, that is the 5th from the South-Westerly. With the Radius of the Rumbs, describe from C, the Circle N E S W. Let S represent the South,

South, and take from the *Line of Rumbs* 5 Points, and lay it from S to R, and draw CR; it is the Rumb required.



2dly. If the Rumb be drawn, describe, as before, a Circle, and apply the Distance R S to the *Line of Rumbs*; it will reach from the Beginning to the 5th. The Use of this may be supplied by the *Line of Chords*, by allowing 11°∟15′ for every Point. The Use of the Meridian is principally, either to draw a *Mercator's* Chart, or to solve the three following Problems.

1st. Both Latitudes, and the Difference of Longitude, being given; to find the Course. From the *Lizard*, in the Latitude of 49° 55′ North, a Ship is bound to *Barbadoes*, in the Latitude of 13°∟10′ North, the Difference of Longitude is 53°. Generally, on the Back of this Meridian Line, runs a Line of equal Parts, on which the Longitude is counted; therefore call it the *Longitude Line*. Extend the Compasses on the Meridian Line,

 from

from 49° 55′, to 13° 10′, and make this Extent a parallel Radius on the Tangents. Then take the Difference of Longitude in your Compasses, and apply it on the Tangents 'till it becomes a Parallel, which you will find to be at 50°: And this is the Ship's Course from the South-west, which is almost S W $\frac{1}{2}$ W.

2dly. Given both Latitudes, as before, and the Course S W $\frac{1}{2}$ W; to find the Difference of Longitude. Extend the Compasses on the Meridian Line, from 49° 55′, to 13° 10′, and make this Extent a parallel Tangent of 45°; then the Parallel of 50° on the Tangents, measured on the Longitude Line, gives 53°, the Difference of Longitude.

3dly. Given one Latitude 49° 55′ North, the Course South 50° Westerly, and the Difference of Longitude 53°; to find the other Longitude. Make 53°, taken from the Longitude Line, a parallel Tangent of the Course 50°; and then the parallel Tangent of 45°, laid on the Meridian Line from 49° 55′ downwards, will reach to 13° 10′, the Latitude sought.

If the Distance taken on the Meridian Line, be measured on the Longitude Line, the Operation will be performed by the artificial Numbers and Tangents very elegantly. So in the first Example, the Distance on the Meridian Line will measure 44° $\frac{1}{2}$. Therefore on the Numbers, the Extent from 44° $\frac{1}{2}$

to

to 53°, will reach on the Tangents from 45° to 50°, the Course. But, lastly, there is no Occasion for this Meridian Line in these Days; for the artificial Tangents counted backwards from 45°, calling every 5 Deg. 10, is really that Meridian Line: This was first demonstrated by that happy Man Dr. *Halley*. But it seems, at first Sight, that we still want the Longitude Line; but this is thus supplied; Take from the artificial Tangents, the Distance from the Radius to the Tangent of $22^\circ\llcorner 40'$: And this Extent being made a Parallel on the *Line of Lines* to 1000, will represent Leagues to measure the Longitude in Leagues by; or made a Parallel at 50°, will represent 50° of Longitude. Then the first of these three Examples wrought by the modern Sector, is thus; Make the Extent from 45°, to $22^\circ\llcorner 40'$ on the artificial Tangents, a Parallel on the Lines at 50°; then the Distance of the two Latitudes (*viz.* 13° $10'$ and $49^\circ\llcorner 55'$, counting as taught above) from the Meridian Line (that is the Tangents) measured on the Lines, will give $44^\circ \frac{1}{2}$. Then on the Numbers, the Extent from $44^\circ \frac{1}{2}$ to 53°, will reach on the Tangents from 45° to 50°, the Course.

By this Time I believe it appears that the Meridian Line was left out with Judgment.

There are three other Cases in Navigation where the Course, Distance, and Difference of Latitude, are concerned: And this is called Sailing by the plain Chart. Any two of these being given, the other may be found;

 and

and that too by the artificial Sines and Numbers, at one Opening of the Compasses; for,

1*st*. The Extent from the Radius, to the Cosine of the Complement of the Course, will reach from the Distance run down, to the Difference of Latitude: Or,

2*dly*. From the Difference of Latitude, up to the Distance. And,

3*dly*. The Extent from the Distance, to the Difference of Latitude on the Numbers, will reach from the Radius, to the Sine of the Complement of the Course.

The following is very ready for Practice, and sufficiently exact in all Cases, when the Navigator casts up his Log every Day, which he ought not to fail of. The Extent from the Sine of the Complement of half the Sum of the Latitudes, to the Radius, will reach from the Departure (that is, the Sum of all the Increments of the Eastings and Westings) to the Difference of Longitude.

From what hath been said, it is abundantly evident, that the Sector is sufficient for all the Calculations necessary for Navigation, particularly if you call to mind how easily and pleasantly the astronomical Problems are solved by it. I mean that of finding the Amplitude and the Azimuth. For the former no more than one Opening of the Compasses is necessary, and the latter but two. The Extent from the Sine Complement of the

the Latitude, to the Radius, reaches from the Sine of the Declination, to the Sine of the Amplitude. And if the Co-Altitude, and Co-Latitude, are two Sides of a spherical Triangle, whose contained Angle is the Azimuth sought. And the third Side is made the $\left\{\begin{matrix}\text{Sum}\\ \text{Diff.}\end{matrix}\right\}$ of 90°, and the Declination when the Latitude and Declination are of $\left\{\begin{matrix}\text{a different}\\ \text{the same}\end{matrix}\right\}$ Kind, you are to work as taught in the 17th Section of the 16th Chap. where are required no more than two Openings of the Compasses.

I could have proceeded to shew the Uses of this admirable Instrument in many more Particulars; as in Dialling, the Calculation of the Hour Distances, at one Opening of the Compasses; the Inscription of the ornamental Furnitures; and, in Astronomy, the Construction of solar Eclipses. In Architecture the Construction of the Catena and Curves, of the more abstruse Kind. In Mensuration, the Management of the most difficult Curves that may be compared with the conick Sections: But these, and several others, must, at present, be laid aside, I not having Leisure, nor Room, here to do it. However, Reader, if you take but as much Pleasure in practising the Directions herein contained, as I did in penning them, you will not repent your reading these few Lines. Farewell.

FINIS.

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5. A

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6. The Practical Surveyor; Shewing, ready and certain Methods for Meaſuring, Mapping, and Adorning all Sorts of Lands and Waters, by the ſeveral Inſtruments uſed for this Purpoſe: Particularly, of a New Theodolite, very convenient to be uſed by thoſe who are reſolved to be Accurate, as well as Expeditious. Together with the Uſe of the ſame Theodolite, in drawing the perſpective Appearance of a Gentleman's Seat, without Meaſuring one ſingle Length, at one ſetting down of the Inſtrument, the Picture having any given Poſition. Alſo, Its Uſe in Levelling, Meaſuring Timber ſtanding; and, by a Sliding Rule improv'd, all Timbers, Shrubs, *&c.* Extracted from the Works of the moſt experienc'd Artiſts, by *John Hammond*.

7. The Deſcription and Uſe of an *Architectonick Sector*: And alſo of the *Architectonick Sliding Plates*, whereby Scales of all Sizes are moſt readily and univerſally obtain'd for Fluting Pillaſters and Columns, and Drawing the Geometrical Planes and Uprights, in any of the Five Orders, according to the given Diameter of a Column. With ſeveral other Scales, very convenient and ready for the Practice of the Ingenious Deſigners of Building. By *T. Carwitham*.

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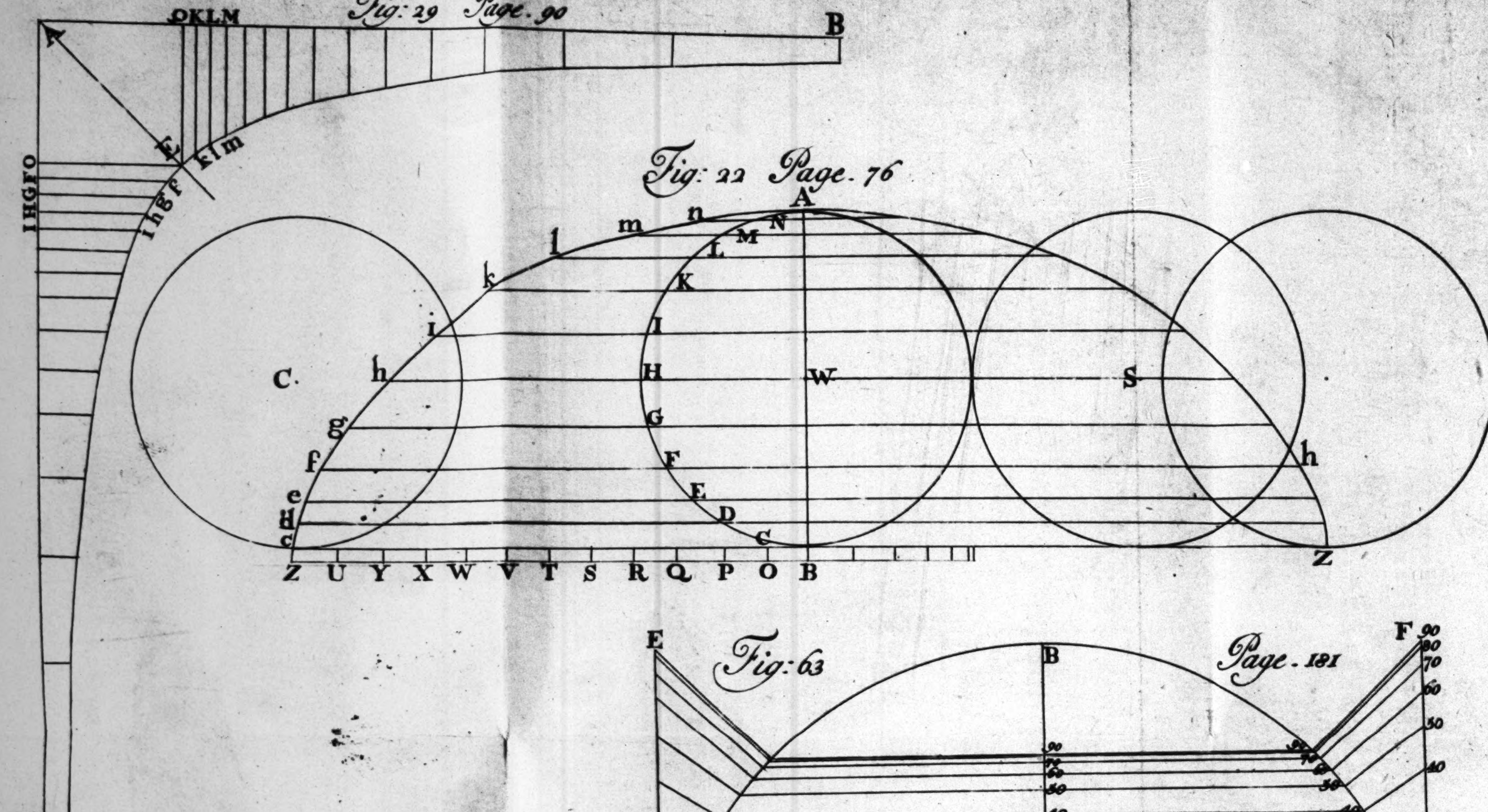

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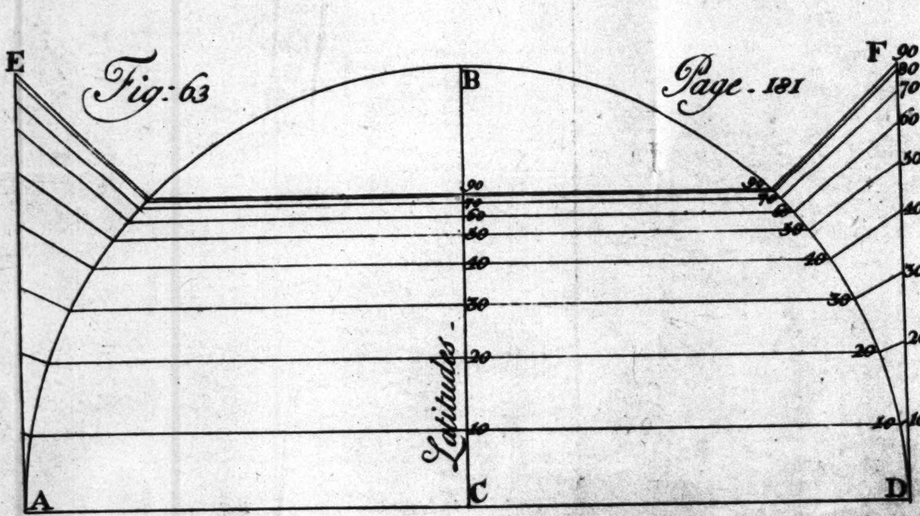

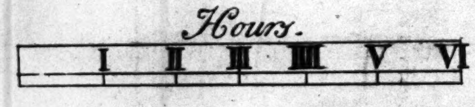

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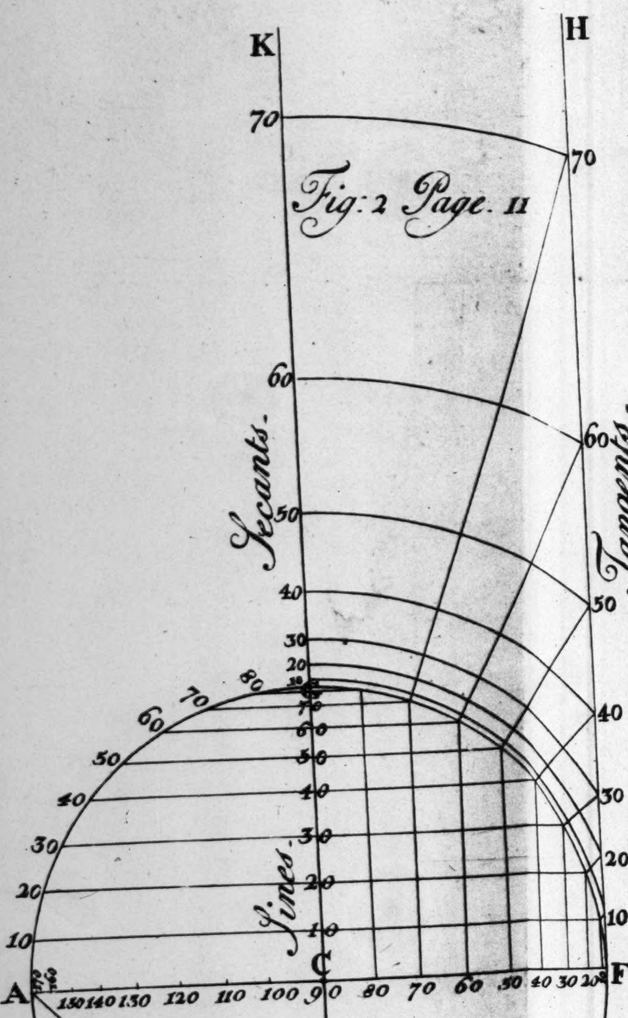

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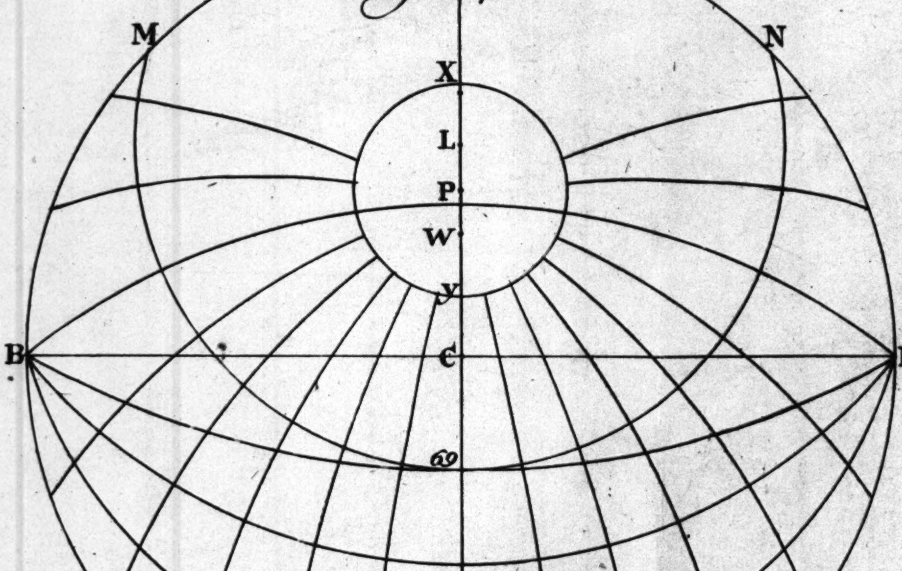
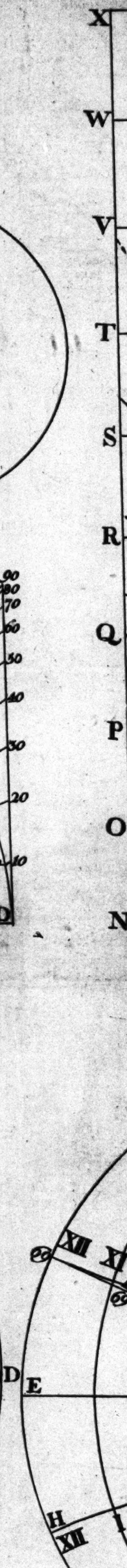
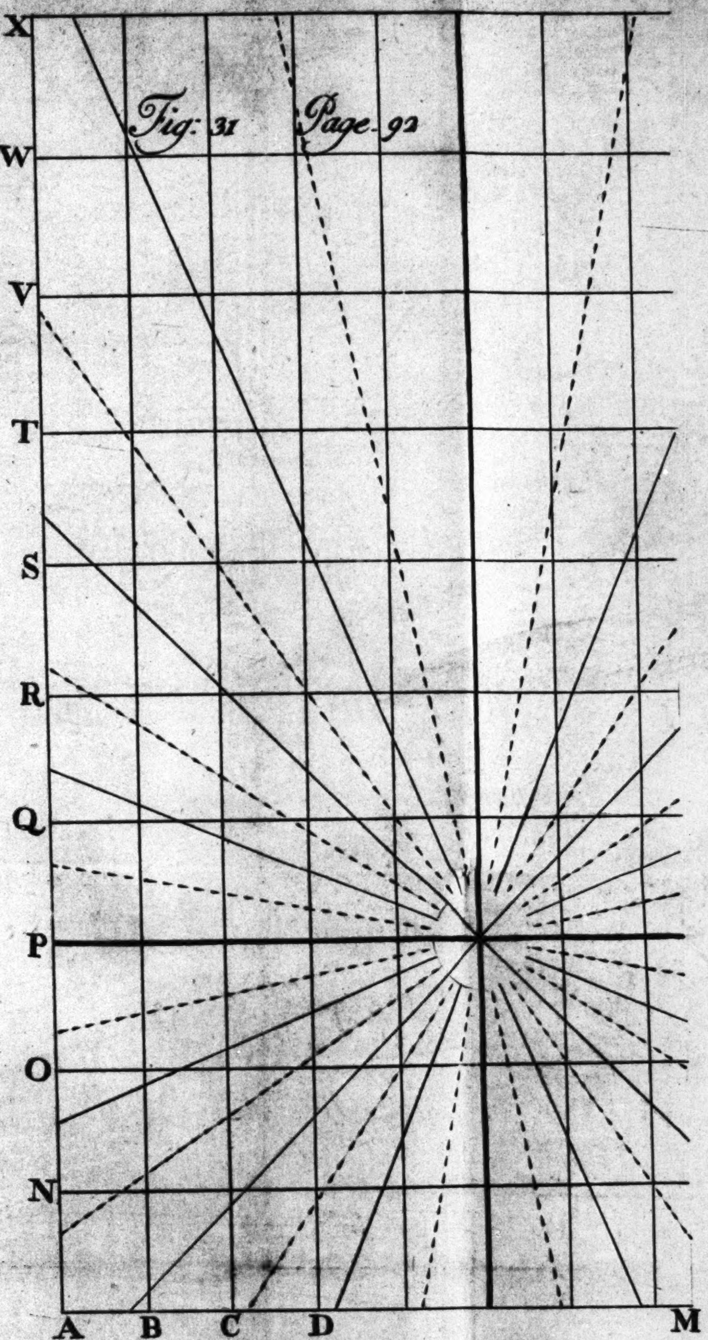


Projection on the Plain of the Horizon.





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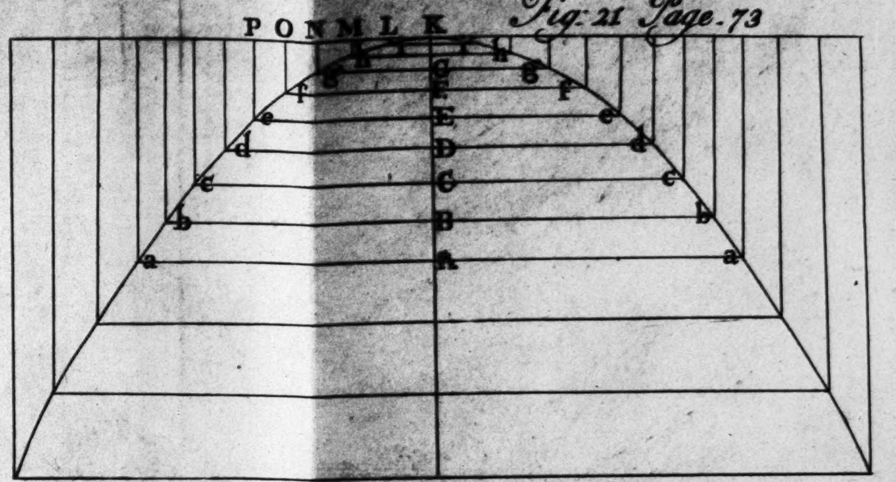

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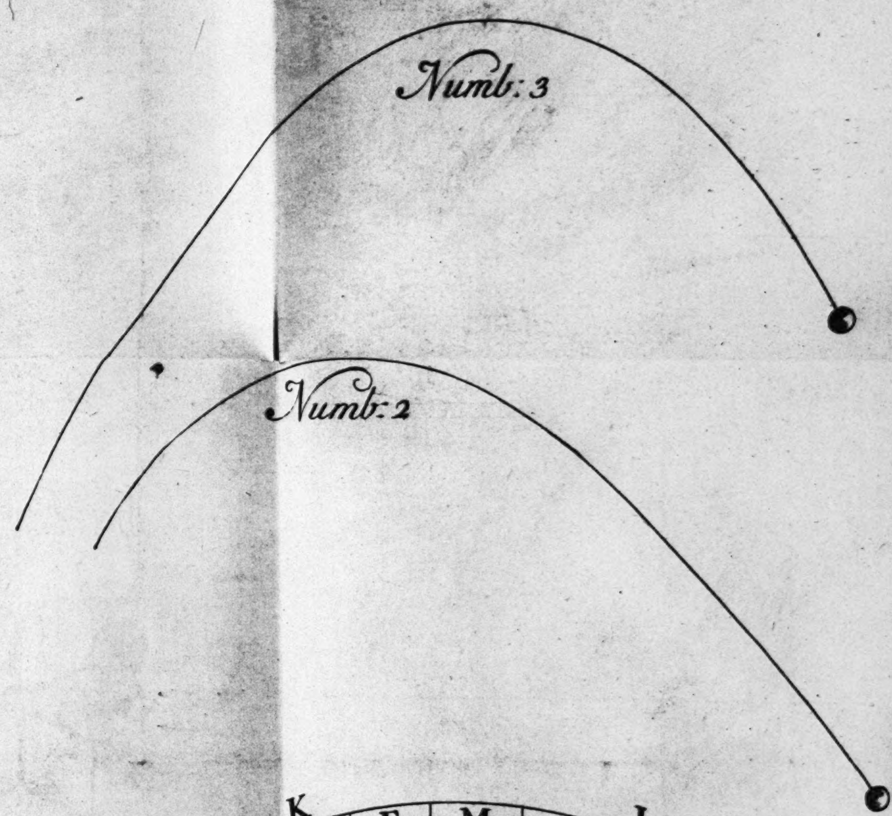

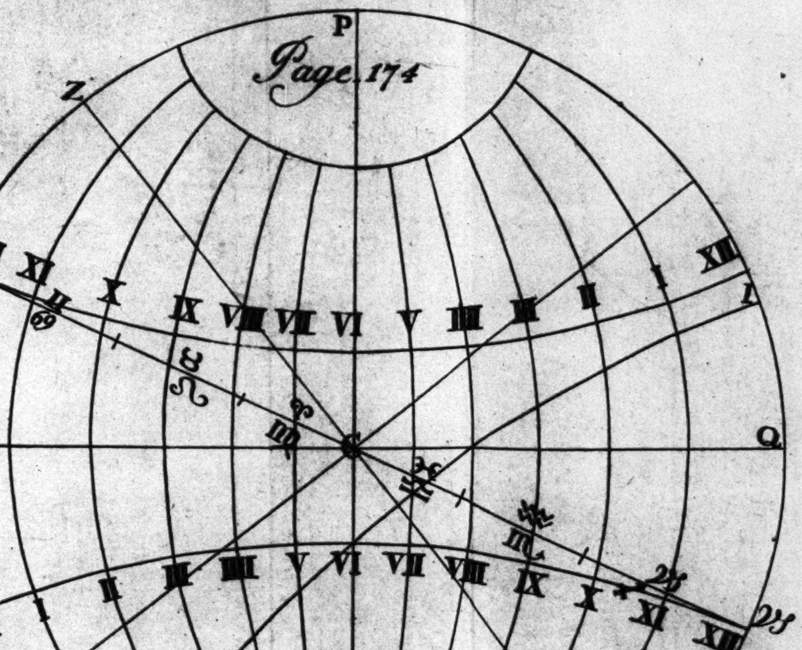

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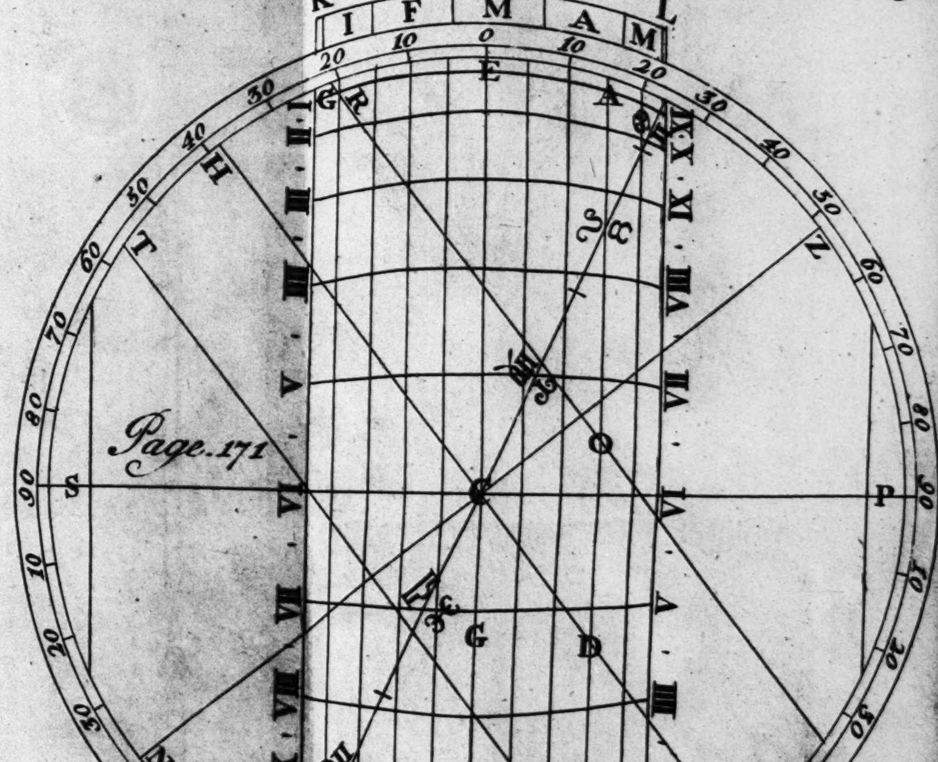
tereographick Projection.



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Orthographick Projection.